

Charge Fractionalization in a Planar System with a Point-Like Vortex Defect at Finite Temperature

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States with fractional charge can appear in quantum systems with topological defects. We consider an ideal gas of two-dimensional Dirac fermions in the background of a point-like magnetic vortex with arbitrary flux and find that this system acquires fractional electric charge at finite temperature. The functional dependence of the thermal average and quadratic fluctuation of the charge on the temperature, the vortex flux, and the continuous parameter of the boundary condition at the location of the vortex defect is determined.

1 Introduction

Spontaneous breakdown of continuous symmetries can give rise to topological defects (texture solitons) with rather interesting properties. A topological defect in three-dimensional space, which is characterized by the nontrivial second homotopy group, is known as a magnetic monopole [1, 2], see also genuine Ref. [3]. Vacuum fluctuations of quantized Dirac fields result in the monopole becoming a CP symmetry violating dyon, i.e. acquiring nonzero (and fractional) electric charge [4–6]. More recently the effect of thermal fluctuations of quantized Dirac fields in the presence of the monopole has been considered, yielding the temperature dependence of the induced charge [7, 8].

A topological defect in two-dimensional space, which is characterized by the nontrivial first homotopy group, is a cross-section of the Abrikosov–Nielsen–Olesen magnetic vortex [9, 10]. The vortex defect is described in terms of a spin-0 field which condenses and a spin-1 field corresponding to the spontaneously broken gauge group; the former is coupled to the latter in the minimal way with constant e_{cond} . Single-valuedness of the condensate field and finiteness of the vortex energy imply that the vortex flux is related to e_{cond} :

$$\Phi = \frac{1}{2\pi} \oint d\mathbf{x} \mathbf{V}(\mathbf{x}) = \frac{1}{e_{\text{cond}}}, \quad (1)$$

where $\mathbf{V}(\mathbf{x})$ is the vector potential of the spin-1 field, and the integral is over a path enclosing once the vortex tube. The quantized fermion field is coupled minimally to the spin-1 field with constant e – the elementary charge; thus, quantum effects depend on the value of $e\Phi$. The case of $e_{\text{cond}} = 2e$ ($e\Phi = 1/2$) is realized in ordinary Bardeen–Cooper–Schrieffer superconductors where the Cooper pair field condenses and, in addition, there are normal electron (pair-breaking) excitations. It remains still to be elucidated, whether other values of $e\Phi$ are realized in nature, although there are claims that vortices with fractional $e\Phi \neq 1/2$ exist in chiral superfluids and chiral and two-gap superconductors [11].

The aim of the present paper is to consider the effect of thermal fluctuations of quantized Dirac fields¹ in the presence of the vortex defect with arbitrary value of $e\Phi$, which results in the vortex acquiring fractional electric charge; the zero-temperature effect was considered earlier [13–15]. Since continuous symmetry is not spontaneously broken at the core of the defect, it seems reasonable to exclude the region of the defect and to impose a boundary condition for quantized fields at the edge of this region. Thus, quantum effects depend both on $e\Phi$ and real continuous quantity Θ which parameterizes the most general varieties of boundary conditions (for more details see next Section). This setup should not be confused with the setup when fermions are quantized in the presence of an extensive magnetic field with finite flux and the region of the nonvanishing field strength is not excluded. The induced charge in the latter case was considered in Refs. [16, 17] (zero temperature) and Ref. [18] (nonzero temperature), and we shall compare the results of both setups in Section 3.

The operator of the second-quantized fermion field in a static background can be presented in the form

$$\Psi(\mathbf{x}, t) = \sum_{E>0} e^{-iEt} \langle \mathbf{x} | E, \lambda \rangle a_{E\lambda} + \sum_{E<0} e^{-iEt} \langle \mathbf{x} | E, \lambda \rangle b_{E\lambda}^+ \quad (2)$$

where $a_{E\lambda}^+$ and $a_{E\lambda}$ ($b_{E\lambda}^+$ and $b_{E\lambda}$) are the fermion (antifermion) creation and destruction operators satisfying anticommutation relations,

$$[a_{E\lambda}, a_{E'\lambda'}^+]_+ = [b_{E\lambda}, b_{E'\lambda'}^+]_+ = \langle E, \lambda | E', \lambda' \rangle, \quad (3)$$

and $\langle \mathbf{x} | E, \lambda \rangle$ is the solution to the stationary Dirac equation,

$$H \langle \mathbf{x} | E, \lambda \rangle = E \langle \mathbf{x} | E, \lambda \rangle, \quad (4)$$

H is the Dirac Hamiltonian, E is the energy and λ is the set of other parameters (quantum numbers) specifying a state; symbol \sum means the summation over discrete and the integration (with a certain measure) over continuous values of all quantum numbers. Conventionally, the operators of dynamical invariants are defined as bilinears of the fermion field operators, and, thus, comprising: the energy operator (temporal component of the energy-momentum vector),

$$\hat{P}^0 = \frac{i}{4} \int d^d x \left([\Psi^+, \partial_t \Psi]_- - [\partial_t \Psi^+, \Psi]_- \right), \quad (5)$$

and the fermion number operator,

$$\hat{N} = \frac{1}{2} \int d^d x [\Psi^+, \Psi]_-, \quad (6)$$

where d is the space dimension. Operators (5) and (6) commute and are thus diagonal in the fermion and antifermion creation and destruction operators.

The thermal average of the fermion number operator over the canonical ensemble is defined as (see, e.g., Ref. [19])

$$\langle \hat{N} \rangle = \frac{\text{Sp } \hat{N} \exp(-\beta \hat{P}^0)}{\text{Sp } \exp(-\beta \hat{P}^0)}, \quad \beta = (k_B T)^{-1}, \quad (7)$$

¹This may be relevant for various particle physics models with applications ranging from early Universe cosmology to hot nuclear matter phenomenology, and even for condensed matter models, because effectively quasirelativistic fermions arise, in particular, in d -wave type II superconductors (see, e.g., Ref. [12]).

where T is the equilibrium temperature, k_B is the Boltzmann constant, and Sp is the trace or the sum over the expectation values in the Fock state basis created by operators in equation (3). Appropriately, the electric charge of the quantum fermionic system in thermal equilibrium is given by expression

$$Q(T) \equiv e\langle\hat{N}\rangle = -\frac{e}{2} \int_{-\infty}^{\infty} dE \tau(E) \tanh\left(\frac{1}{2}\beta E\right), \quad (8)$$

where the last equality is obtained by transforming the right-hand side of equation (7) into an integral over the spectrum of the Dirac Hamiltonian (see, e.g., Ref. [18]), and the spectral density of the Dirac Hamiltonian (or density of states) is

$$\tau(E) = \frac{1}{\pi} \text{Im Tr} \frac{1}{H - E - i0}, \quad (9)$$

where Tr is the trace of an integro-differential operator in functional space:

$$\text{Tr} U = \int d^d x \text{tr} \langle \mathbf{x} | U | \mathbf{x} \rangle;$$

tr denotes the trace over spinor indices only; note that the functional trace should be regularized and renormalized by subtraction, if necessary.

Similarly, one gets expression for the quadratic fluctuation of the electric charge:

$$\Delta_{Q(T)}^2 \equiv e^2 \left[\langle \hat{N}^2 \rangle - (\langle \hat{N} \rangle)^2 \right] = \frac{e^2}{4} \int_{-\infty}^{\infty} dE \frac{\tau(E)}{\cosh^2\left(\frac{1}{2}\beta E\right)}. \quad (10)$$

Evidently, if the quadratic fluctuation becomes nonvanishing, then the corresponding dynamical invariant ceases to be a sharp quantum observable.

In the present paper we shall find electric charge (8) and its fluctuation (10) in the $d = 2$ quantum fermionic system in the background of a single static topological defect which is a two-dimensional cross section of the magnetic vortex.

2 Thermal average and fluctuation of the charge

Taking into account equation (9), one can get the following contour integral representation for induced charge (8) and its quadratic fluctuation (10):

$$Q(T) = -\frac{e}{2} \int_C \frac{d\omega}{2\pi i} \tanh\left(\frac{1}{2}\beta\omega\right) \text{Tr} (H - \omega)^{-1}, \quad (11)$$

and

$$\Delta_{Q(T)}^2 = \frac{e^2}{4} \int_C \frac{d\omega}{2\pi i} \text{sech}^2\left(\frac{1}{2}\beta\omega\right) \text{Tr} (H - \omega)^{-1}, \quad (12)$$

where C is the contour $(-\infty + i0, +\infty + i0)$ and $(+\infty - i0, -\infty - i0)$ in the complex ω -plane.

Thus we obtain:

$$Q(T) = -\frac{e}{2} \text{sgn}(m) \left\{ \frac{1}{2} [\text{sgn}(1 + A^{-1}) - \text{sgn}(1 + A)] \tanh\left(\frac{1}{2}\beta|E_{BS}|\right) + \frac{2 \sin(F\pi)}{\pi} \int_0^\infty \frac{du}{u\sqrt{u+1}} \tanh\left(\frac{1}{2}\beta|m|\sqrt{u+1}\right) \times \frac{FAu^F - (1-F)A^{-1}u^{1-F} - u \cos(F\pi) + (F - \frac{1}{2})u(Au^F + A^{-1}u^{1-F})}{[Au^F - A^{-1}u^{1-F} + 2 \cos(F\pi)]^2 + 4(u+1) \sin^2(F\pi)} \right\}, \quad (13)$$

and

$$\begin{aligned} \Delta_{Q(T)}^2 = & \frac{e^2}{4} \left\{ \frac{1}{2} [1 - \operatorname{sgn}(A)] \operatorname{sech}^2 \left(\frac{1}{2} \beta |E_{BS}| \right) - F(1-F) \operatorname{sech}^2 \left(\frac{1}{2} \beta |m| \right) \right. \\ & + \frac{2 \sin(F\pi)}{\pi} \int_0^\infty \frac{du}{u} \operatorname{sech}^2 \left(\frac{1}{2} \beta |m| \sqrt{u+1} \right) \\ & \left. \times \frac{FAu^F + (1-F)A^{-1}u^{1-F} - (2F-1)u \cos(F\pi)}{[Au^F - A^{-1}u^{1-F} + 2 \cos(F\pi)]^2 + 4(u+1) \sin^2(F\pi)} \right\}, \end{aligned} \quad (14)$$

where

$$F = e\Phi - \llbracket e\Phi \rrbracket, \quad 0 \leq F < 1, \quad (15)$$

is the fractional part of $e\Phi$,

$$A = 2^{1-2F} \frac{\Gamma(1-F)}{\Gamma(F)} \tan \left(s \frac{\Theta}{2} + \frac{\pi}{4} \right), \quad (16)$$

and Θ is the continuous real parameter of the boundary condition at the location of the vortex defect; recall that bound state energy E_{BS} is determined implicitly by [15]

$$\frac{(1 + m^{-1}E_{BS})^{1-F}}{(1 - m^{-1}E_{BS})^F} = -A. \quad (17)$$

In the cases of $A = 0$ and $A^{-1} = 0$ expressions for the charge and its fluctuation simplify:

$$Q(T) = -\frac{e}{2} \left(F - \frac{1}{2} \pm \frac{1}{2} \right) \tanh \left(\frac{1}{2} \beta m \right), \quad \Theta = \pm s \frac{\pi}{2} \pmod{2\pi}, \quad (18)$$

and

$$\Delta_{Q(T)}^2 = \frac{e^2}{4} \left(F - \frac{1}{2} \pm \frac{1}{2} \right)^2 \operatorname{sech}^2 \left(\frac{1}{2} \beta |m| \right), \quad \Theta = \pm s \frac{\pi}{2} \pmod{2\pi}; \quad (19)$$

note that equation (18) at $\Theta = s \frac{\pi}{2} \pmod{2\pi}$ was obtained in Ref. [7].

In the limit $T \rightarrow 0$ ($\beta \rightarrow \infty$) the charge tends to finite value (see Ref. [15]):

$$Q(0) = \begin{cases} \left. \begin{aligned} & \frac{e}{2} \operatorname{sgn}(m)(1-F), & -1 < A < \infty \\ & -\frac{e}{2} \operatorname{sgn}(m)F, & A^{-1} = -1, A^{-1} = 0 \\ & -\frac{e}{2} \operatorname{sgn}(m)(1+F), & -\infty < A < -1 \end{aligned} \right\}, & 0 < F < \frac{1}{2}, \end{cases} \quad (20)$$

$$Q(0) = \begin{cases} \left. \begin{aligned} & -\frac{e}{2} \operatorname{sgn}(m)F, & -1 < A^{-1} < \infty \\ & \frac{e}{2} \operatorname{sgn}(m)(1-F), & A = -1, A = 0 \\ & \frac{e}{2} \operatorname{sgn}(m)(2-F), & -\infty < A^{-1} < -1 \end{aligned} \right\}, & \frac{1}{2} < F < 1, \end{cases}$$

$$Q(0) = \begin{cases} \left. \begin{aligned} & -\frac{e}{\pi} s \operatorname{sgn}(m) \arctan \left(\tan \frac{\Theta}{2} \right), & \Theta \neq \pi \pmod{2\pi} \\ & 0, & \Theta = \pi \pmod{2\pi} \end{aligned} \right\}, & F = \frac{1}{2}, \end{cases} \quad (21)$$

whereas the fluctuation tends exponentially to zero for almost all values of Θ with the exception of one corresponding to the zero bound state energy, $E_{BS} = 0$ ($A = -1$):

$$\Delta_{Q(0)}^2 = \begin{cases} 0, & A \neq -1, \\ \frac{e^2}{4}, & A = -1. \end{cases} \quad (22)$$

In the high-temperature limit the charge tends to zero:

$$Q(T \rightarrow \infty) = \begin{cases} \frac{e}{2} \operatorname{sgn}(m) \frac{\sin(F\pi)}{\pi} \frac{\Gamma(1-F)}{\Gamma(1+F)} \tan\left(s\frac{\Theta}{2} + \frac{\pi}{4}\right) \left(\frac{|m|}{k_B T}\right)^{1-2F}, & 0 < F < \frac{1}{2}, \\ -\frac{e}{8} \frac{sm}{k_B T} \sin \Theta, & F = \frac{1}{2}, \\ -\frac{e}{2} \operatorname{sgn}(m) \frac{\sin(F\pi)}{\pi} \frac{\Gamma(F)}{\Gamma(2-F)} \cot\left(s\frac{\Theta}{2} + \frac{\pi}{4}\right) \left(\frac{|m|}{k_B T}\right)^{2F-1}, & \frac{1}{2} < F < 1. \end{cases} \quad (23)$$

whereas the fluctuation tends to finite value:

$$\lim_{T \rightarrow \infty} \Delta_{Q(T)}^2 = \begin{cases} \left. \begin{aligned} &\frac{e^2}{4}(1-F)^2, & \Theta \neq s\frac{\pi}{2} \pmod{2\pi} \\ &\frac{e^2}{4}F^2, & \Theta = s\frac{\pi}{2} \pmod{2\pi} \end{aligned} \right\}, & 0 < F \leq \frac{1}{2}, \\ \left. \begin{aligned} &\frac{e^2}{4}F^2, & \Theta \neq -s\frac{\pi}{2} \pmod{2\pi} \\ &\frac{e^2}{4}(1-F)^2, & \Theta = -s\frac{\pi}{2} \pmod{2\pi} \end{aligned} \right\}, & \frac{1}{2} \leq F < 1. \end{cases} \quad (24)$$

At half-integer values of $e\Phi$ one has

$$A|_{F=\frac{1}{2}} = \tan\left(s\frac{\Theta}{2} + \frac{\pi}{4}\right), \quad (25)$$

and the charge and its fluctuation take the form

$$Q(T)|_{F=\frac{1}{2}} = -\frac{e}{4}s \left\{ [1 - \operatorname{sgn}(\cos \Theta)] \tanh\left(\frac{1}{2}\beta m \sin \Theta\right) + \frac{\sin 2\Theta}{2\pi} \int_1^\infty \frac{dv}{\sqrt{v(v-1)}} \frac{\tanh\left(\frac{1}{2}\beta m \sqrt{v}\right)}{v - \sin^2 \Theta} \right\}, \quad (26)$$

and

$$\Delta_{Q(T)}^2|_{F=\frac{1}{2}} = \frac{e^2}{8} \left\{ [1 - \operatorname{sgn}(\cos \Theta)] \operatorname{sech}^2\left(\frac{1}{2}\beta|m \sin \Theta|\right) - \frac{1}{2} \operatorname{sech}^2\left(\frac{1}{2}\beta|m|\right) + \frac{\cos \Theta}{\pi} \int_1^\infty \frac{dv}{\sqrt{v-1}} \frac{\operatorname{sech}^2\left(\frac{1}{2}\beta|m|\sqrt{v}\right)}{v - \sin^2 \Theta} \right\}. \quad (27)$$

An alternative representation for the charge and its fluctuation is obtained by deforming contour C to encircle poles of the $\tanh\left(\frac{1}{2}\beta\omega\right)$ and $\operatorname{sech}^2\left(\frac{1}{2}\beta\omega\right)$ functions, which occur along the imaginary axis at the Matsubara modes ($\omega_n = (2n+1)\frac{i\pi}{\beta}$):

$$Q(T) = -e \operatorname{sgn}(m) \left\{ \frac{1}{2} \left(F - \frac{1}{2}\right) \tanh\left(\frac{\pi}{2\xi}\right) \right. \quad (28)$$

$$+ \frac{\xi}{\pi} \sum_{\substack{n \in \mathbb{Z} \\ n \geq 0}} \frac{2(2F-1)(2n+1)^2 \xi^2 + A[1 + (2n+1)^2 \xi^2]^F - A^{-1}[1 + (2n+1)^2 \xi^2]^{1-F}}{[1 + (2n+1)^2 \xi^2] \{A[1 + (2n+1)^2 \xi^2]^F + 2 + A^{-1}[1 + (2n+1)^2 \xi^2]^{1-F}\}},$$

and

$$\begin{aligned} \Delta_{Q(T)}^2 &= \frac{e^2}{8} [1 - 2F(1-F)] \operatorname{sech}^2 \left(\frac{\pi}{2\xi} \right) \\ &+ e^2 \frac{\xi^2}{\pi^2} \sum_{\substack{n \in \mathbb{Z} \\ n \geq 0}} \frac{1}{[1 + (2n+1)^2 \xi^2]^2 \{A[1 + (2n+1)^2 \xi^2]^F + 2 + A^{-1}[1 + (2n+1)^2 \xi^2]^{1-F}\}} \\ &\times \left\{ (2F-1)[(2n+1)^2 \xi^2 - 1] \{A[1 + (2n+1)^2 \xi^2]^F - A^{-1}[1 + (2n+1)^2 \xi^2]^{1-F}\} \right. \\ &+ 2\{1 - [3 - 4F(1-F)](2n+1)^2 \xi^2\} - 4(2n+1)^2 \xi^2 \\ &\left. \times \frac{(2F-1)\{A[1 + (2n+1)^2 \xi^2]^F - A^{-1}[1 + (2n+1)^2 \xi^2]^{1-F}\} - 1 + (2F-1)^2(2n+1)^2 \xi^2}{A[1 + (2n+1)^2 \xi^2]^F + 2 + A^{-1}[1 + (2n+1)^2 \xi^2]^{1-F}} \right\}, \end{aligned} \quad (29)$$

where $\xi = \pi/(\beta|m|)$.

3 Discussion

In the present paper we consider an ideal gas of two-dimensional relativistic massive electrons in the background of a static point-like magnetic vortex. This system at thermal equilibrium is found to acquire electric charge: its average $Q(T)$ is given by equation (13), and its quadratic fluctuation $\Delta_{Q(T)}^2$ is given by equation (14). The most general boundary conditions (parametrized by the self-adjoint extension parameter Θ) at the location of the vortex are employed, and arbitrary values of the vortex flux Φ are permitted; our results are periodic in Θ with period 2π at fixed Φ and periodic in Φ with period e^{-1} at fixed Θ (e is the electron charge). Note that equations (13) and (14) can be regarded as the Sommerfeld–Watson transforms of the infinite sum representation, equations (28) and (29). Note also that the charge is odd and its fluctuation is even under transition to the inequivalent representation of the Clifford algebra ($m \rightarrow -m$).

Equation (13) can be rewritten in the form

$$Q(T) = Q(0) + \tilde{Q}(T), \quad (30)$$

where $Q(0)$ is given by equations (20)–(21) [15], and

$$\begin{aligned} \tilde{Q}(T) &= \frac{e}{2} \operatorname{sgn}(m) \left\{ \frac{\operatorname{sgn}(1 + A^{-1}) - \operatorname{sgn}(1 + A)}{\exp(\beta|E_{BS}|) + 1} + \frac{2F-1}{\exp(\beta|m|) + 1} \right. \\ &\left. + \frac{\beta|m|}{2\pi} \int_1^\infty dw \operatorname{sech}^2 \left(\frac{1}{2}\beta|m|w \right) \arctan \left[\frac{A(w^2 - 1)^F - A^{-1}(w^2 - 1)^{1-F} + 2 \cos(F\pi)}{2w \sin(F\pi)} \right] \right\}. \end{aligned} \quad (31)$$

Our result should be compared with the result of Ref. [18]

$$Q(T) = -\frac{e^2}{2} s\Phi \tanh \left(\frac{1}{2}\beta m \right), \quad (32)$$

where Φ is the flux of a magnetic field with an extensive support, and it is implied that the region of the support is not excluded. Thus, result (32) describes the direct effect of the field

strength, whereas our result describes the indirect, through the vector potential, effect of the field strength from the excluded region. In contrast to equation (32), our expressions for $Q(T)$ and $\Delta_{Q(T)}^2$ are periodic in the value of the flux, vanishing at integer values of $e\Phi$, and this can be regarded as a manifestation of the Bohm–Aharonov effect [20] in quantum field theory at nonzero temperature.

The nonvanishing of the charge quadratic fluctuation signifies that the charge of the system is not a sharp quantum observable and has to be understood as a thermal expectation value only. In the high-temperature limit the average charge tends to zero (23) and the fluctuation tends to finite value (24). In the zero-temperature limit quantities $\Delta_{Q(T)}^2$ and $\tilde{Q}(T)$ tend exponentially to zero and the charge becomes a sharp quantum observable with finite value $Q(0)$ (20)–(21). However, the last statement is true for almost all values of Θ with the exception of one corresponding to the zero bound state energy, $E_{BS} = 0$ ($A = -1$), since in this case the zero-temperature fluctuation is nonzero, see equation (22).

At half-integer values of $e\Phi$ the average charge takes form of equation (26) which coincides (after substituting s for $2eg$, where g is the magnetic monopole charge, $2eg = n$ is the Dirac quantization condition) with the expression for the thermally induced charge in the monopole background in three-dimensional space [7, 8]. It should be emphasized that at non-half-integer values of $e\Phi$ the behavior of the charge as a function of Θ differs drastically from the one at half-integer $e\Phi$.

In the $F \neq \frac{1}{2}$ case the charge at zero temperature is given by a step function with two jumps. As temperature increases, the jump corresponding to the zero bound state energy ($A = -1$) is smoothed out, while another jump is persisting. The charge at $A = -1$ is not a sharp quantum observable even at zero temperature, which is explicated by the nonvanishing of the fluctuation in this case. As temperature departs from zero, the fluctuation develops a maximum at $A = -1$ and a minimum close to the position of the persisting jump of the charge, but out of the region where bound state exists. With the increase of temperature the maximum is widening and disappearing, while the minimum is narrowing with its position approaching the position of the charge jump and its width tending to zero in the high-temperature limit.

In the $F = \frac{1}{2}$ case the charge at zero temperature is linear in Θ with one jump at $A = -1$ ($\Theta = s\pi \pmod{2\pi}$) where the charge is not a sharp quantum observable. As temperature increases, this jump is smoothed out. Appropriately, the fluctuation is symmetric with respect to the position of this jump, and a maximum of the fluctuation is smoothed out with the increase of temperature.

In conclusion we note that the system considered can acquire, in addition to the charge, also other quantum numbers. In the case of zero temperature this issue is comprehensively elucidated in Refs. [15, 21, 22], and an appropriate generalization to the case of nonzero temperature will be studied elsewhere.

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