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# Symmetries in Semiclassical Mechanics

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Symmetry properties in quantum, classical and semiclassical mechanics are considered. Geometric structures in the semiclassical (Maslov complex-WKB) theory are investigated. Infinitesimal properties of semiclassical symmetry transformations are discussed.

## 1 Introduction

Semiclassical approximation is widely used in quantum mechanics and field theory. Many different approaches to constructing semiclassical theories (functional integral approach, operator approach etc.) have been developed. However, the most appropriate for mathematical justification approach is based on direct substitution of the wave function to the quantum evolution equation.

Historically, the WKB wave function was the first substitution that obeyed the Schrödinger equation in the semiclassical approximation. Now, many different semiclassical substitutions (arised in the Maslov theory of wave packets, in the Maslov theory of Lagrangian manifolds with complex germs etc) to the Schrödinger equation are known. The simplest one is the wave packet function which is specified by classical variables (numbers: $S$ – phase, $P_i$ – momenta, $Q_i$ – coordinates) and quantum function $f$ specifying a shape of the wave packet.

Generally, a semiclassical theory may be viewed as follows. A semiclassical state is a point on the space of a bundle ("semiclassical bundle"), which can be denoted as $(X, f)$, where $X$ is a classical state (for ordinary quantum mechanics, $X = (S, P, Q)$ and $f$ is a quantum state in the given classical background). Set of all $\{X\}$ may be considered as a base of the bundle, while $f$ belongs to a Hilbert space $\mathcal{F}_X$, which may be generally $X$-dependent.

If the quantum theory model is symmetric under a Lie group, the corresponding semiclassical theory should be also symmetric. This means that the symmetry Lie group acts on the semiclassical bundle; an automorphism of the bundle corresponds to each element of the Lie group and the group property is satisfied.

When one is interested whether the quantum theory is symmetric under a Lie group, provided that the corresponding classical theory is symmetric, a first step to solve the problem is to investigate the corresponding semiclassical theory. Quantum anomalies may be investigated even in the semiclassical level.

The purpose of the talk is to investigate the properties of the semiclassical mechanics symmetric under Lie groups. The following problems are to be discussed: infinitesimal properties; correspondence between Lie groups and algebras in the semiclassical mechanics; properties of semiclassical gauge theories; applications to quantum field theory.

## 2 Symmetries in classical and quantum mechanics

There are many classical and quantum systems with symmetries. The simplest example of the symmetry transformation is evolution. Group of Poincaré transformations is also a typical example of symmetry group. More complicated symmetries (such as gauge) are also known.

Although the mathematical formalism of quantum mechanics differs from the classical one, and the spaces of states are quite different, the symmetry notions are similar. For example, the evolution is viewed as a transformation mapping the initial state to the state at time $t$. In quantum mechanics (see, for example, [3]) it is an unitary operator $\hat{U}_t = e^{-i\hat{H}t}$ acting in the Hilbert space $\mathcal{H}$ (quantum state space); classically, it is a symplectic transformation of the classical state space (phase space) generated by a Hamiltonian vector field (see, for example, [1]).

The comparison of classical and quantum symmetries is presented in Table 1.

**Table 1.** Symmetries in classical and quantum mechanics.

| | Classical mechanics | Quantum mechanics |
|---|---|---|
| State space | Symplectic manifold $\mathcal{M}$ ($X = (p, q)$) | Hilbert space $\mathcal{H}$ |
| Symmetry transformation under Lie group $G$ | $g \in G \mapsto u_g : \mathcal{M} \to \mathcal{M}$ is smooth symplectic transformation; $u_{g_1 g_2} = u_{g_1} u_{g_2}$; $u_e = 1$ | $g \in G \mapsto \hat{U}_g : \mathcal{H} \to \mathcal{H}$ is unitary operator; $\hat{U}_{g_1 g_2} = \hat{U}_{g_1} \hat{U}_{g_2}$; $\hat{U}_e = 1$ |
| Constructing transformations for $g(t) = e^{t\delta g}$, $\delta g$ belongs to the Lie algebra | $u_{g(t)}$ takes the initial condition for equation $$\frac{dX(t)}{dt} = \nabla[\delta g](X(t)), \quad X(t) \in \mathcal{M}$$ to the solution of the Cauchy problem: $u_{g(t)} : X(0) \mapsto X(t)$; $\nabla[\delta g](X)$ is Hamiltonian vector field on $\mathcal{M}$ | $\hat{U}_{g(t)} = \exp[-i\hat{H}[\delta g]t]$ takes the initial condition for equation $$i\frac{d\psi}{dt} = \hat{H}[\delta g]\psi, \quad \psi(t) \in \mathcal{H}$$ to the solution of the Cauchy problem $\hat{U}_{g(t)} : \psi(0) \mapsto \psi(t)$; $\hat{H}[\delta g]$ is self-adjoint operator in $\mathcal{H}$ |
| Properties of infinitesimal generators | For operator $\nabla[\delta g] = (\nabla[\delta g])^i \frac{\partial}{\partial X^i}$, $$[\nabla[\delta g_1]; \nabla[\delta g_2]] = -\nabla[\delta g_1; \delta g_2]$$ | $[\hat{H}[\delta g_1]; \hat{H}[\delta g_2]] = -i\hat{H}[\delta g_1; \delta g_2]$ |

## 3 What is semiclassical approximation?

It follows from Table 1 that main notions of classical and quantum mechanics are different. However, both theories describe nature. They should not contradict each other then; quantum systems should be interpreted from the classical viewpoint under certain conditions.

Such a correspondence is possible, provided that quantum theory depends on the parameter $h$, and $h \to 0$. This is a "small parameter" of the semiclassical expansion. It is proportional to the Planck constant. The dependence of self-adjoint generator $\hat{H}$ of the symmetry transformation $e^{-i\hat{H}t}$ on $h$ should be as follows:

$$\hat{H}_h = \frac{1}{h} H\left(x, -ih\frac{\partial}{\partial x}\right), \qquad x \in \mathbb{R}^n.$$

The definition of function of non-commuting operators $x$ and $-ih\frac{\partial}{\partial x}$ can be found in [4].

Therefore, the equation to be investigated as $h \to 0$ reads

$$ih\frac{\partial \psi^t(x)}{\partial t} = H\left(x, -ih\frac{\partial}{\partial x}\right)\psi^t(x). \tag{1}$$

Historically, the first approximate solution of equation (1) was (see, for example, [3]) the WKB solution of the form

$$\psi^t(x) = \varphi^t(x) e^{\frac{i}{h}S^t(x)}. \tag{2}$$

Substituting expression (2) to equation (1), one obtains the Hamilton–Jacobi equation for $S^t(x)$ and the transport equation for $\varphi^t(x)$.

Now, a lot of approximate solutions for equation (1) are known. For example, there is the Maslov substitution [4, 5] of the form:

$$\psi^t(x) = \mathrm{const}\, e^{\frac{i}{h}S^t} e^{\frac{i}{h}P^t(x-Q^t)} f^t\left(\frac{x-Q^t}{\sqrt{h}}\right) \equiv (K^h_{S^t,P^t,Q^t} f^t)(x), \tag{3}$$

with $S^t \in \mathbb{R}$, $P^t \in \mathbb{R}^n$, $Q^t \in \mathbb{R}^n$, $f^t \in S(\mathbb{R}^n)$. The normalizing factor should be chosen to be const $\sim h^{-n/4}$ in order to make the norm $\|\psi^t\|$ to be of the order $O(1)$.

One finds by the direct calculation that the wave packet $\psi^t$ approximately satisfies equation (1) under conditions

$$\frac{dS^t}{dt} = P^t \frac{dQ^t}{dt} - H(Q^t, P^t), \tag{4}$$

$$\frac{dQ^t_j}{dt} = \frac{\partial H}{\partial P_j}(Q^t, P^t), \qquad \frac{dP^t_j}{dt} = -\frac{\partial H}{\partial Q_j}(Q^t, P^t), \tag{5}$$

$$i\frac{\partial f^t(\xi)}{\partial t} = \sum_{js}\left[\frac{1}{2}\frac{1}{i}\frac{\partial}{\partial \xi_j}\frac{\partial^2 H}{\partial P_j \partial P_s}\frac{1}{i}\frac{\partial}{\partial \xi_s} + \xi_j \frac{\partial^2 H}{\partial Q_j \partial P_s}\frac{1}{i}\frac{\partial}{\partial \xi_s} + \frac{1}{2}\xi_j \frac{\partial^2 H}{\partial P_j \partial P_s}\xi_s\right] f^t(\xi). \tag{6}$$

Let us compare the substitutions (2) and (3) at fixed time moment $t$. One can notice that the width of the WKB wave function is of the order $O(1)$; the same conclusion is valid for its Fourier transformation as well. On the other hand, the Maslov wave packet (3), as well as its Fourier transformation, is of the width $O(\sqrt{h})$ being small as $h \to 0$. This means that the uncertainties of coordinates and momenta are of the order $O(\sqrt{h})$. The quantum state (3) can be interpreted as a classical particle with coordinates $Q^t \in \mathbb{R}^n$ and momenta $P^t \in \mathbb{R}^n$ then.

It is remarkable that all known semiclassical solutions (see [4]) of equation (1) (including WKB) can be constructed [6] as superpositions of the Maslov wave packets (3):

$$\psi(x) = \mathrm{const}\int d\alpha e^{\frac{i}{h}S(\alpha)} e^{\frac{i}{h}P(\alpha)(x-Q(\alpha))} f\left(\alpha, \frac{x-Q(\alpha)}{\sqrt{h}}\right), \qquad \alpha \in \mathbb{R}^k. \tag{7}$$

It happens that the state (7) is nontrivial, only if the Maslov isotropic condition

$$\frac{\partial S}{\partial \alpha_a} = \sum_j P_j \frac{\partial Q_j}{\partial \alpha_a} \tag{8}$$

is satisfied. Otherwise,

$$\|\psi\| = O(h^\infty).$$

Under condition (8), one should choose const $\sim h^{-\frac{n+k}{4}}$, and

$$(\psi, \psi) \simeq \int d\alpha d\xi f^*(\alpha, \xi) \prod_a 2\pi\delta\left(\sum_j \left\{\frac{\partial P_j}{\partial \alpha_a}\xi_j - \frac{\partial Q_j}{\partial \alpha_a}\frac{1}{i}\frac{\partial}{\partial \xi_j}\right\}\right) f(\alpha, \xi). \tag{9}$$

Notice also that the wave function (7) is not small only in the vicinity (of the width $\sqrt{h}$) around the $k$-dimensional surface $\{Q(\alpha)\}$.

## 4 Geometry of semiclassical mechanics

The wave-packet semiclassical solutions (3) can be interpreted geometrically as follows [7]. First of all, notice that the state $K^h_{S,P,Q} f$ can be viewed as a point on a bundle ("semiclassical bundle"). Its base $\mathcal{X}$ is a classical phase space extended by adding a new variable $S$; it is $\mathcal{X} = \mathbb{R}^{2n+1} = \{X = (S, P, Q)\}$. All fibres $\mathcal{F}_X = L^2(\mathbb{R}^n) = \{f\}$ being Hilbert spaces of quantum states in a given classical background $X \in \mathcal{X}$ are identical.

The evolution transformation takes the initial data $(X^0 = (S^0, P^0, Q^0), f^0)$ to the solution of the Cauchy problem for system of equations (4)–(6) $(X^t = (S^t, P^t, Q^t), f^t)$:

$$X^t = u_t X^0, \qquad f^t = U_t(X^t \leftarrow X^0) f^0.$$

It is an automorphism of the semiclassical bundle, since the evolution of $X$ does not depend on $f$.

Classical geometric structures appear in the semiclassical theory as follows. The wave functions

$$K^h_{X^t} f^t \equiv K^h_{S^t, P^t, Q^t} f^t, \tag{10}$$

$$K^h_{X^t + h\delta X^t}(f^t + h\delta f^t) \equiv K^h_{S^t + h\delta S^t, P^t + h\delta P^t, Q^t + h\delta Q^t}(f^t + h\delta f^t) \tag{11}$$

should both be approximate solutions of equation (1) as $h \to 0$, provided that $(X^t + h\delta X^t, f^t + h\delta f^t)$ satisfies the system (4)–(6). However, as $h \to 0$,

$$K^h_{X^t + h\delta X^t}(f^t + h\delta f^t) \simeq e^{-i\omega_{X^t}[\delta X^t]} K^h_{X^t} f^t, \tag{12}$$

with

$$\omega_X[\delta X] = \sum_j P_j \delta Q_j - \delta S, \tag{13}$$

so that both functions (10) and (12) may be approximate solutions only if

$$\omega_{X^t}[\delta X^t] = \sum_j P^t_j \delta Q^t_j - \delta S^t = \text{const}. \tag{14}$$

Analogously, one finds

$$K^h_{X^t + \sqrt{h}\delta X^t}(f^t + \sqrt{h}\delta f^t) = \text{const}\, K^h_{X^t} e^{i\Omega_{X^t}[\delta X^t]} f^t, \tag{15}$$

with

$$(\Omega_X[\delta X] f)(\xi) = \sum_j \left( \delta P_j \xi_j - \delta Q_j \frac{1}{i} \frac{\partial}{\partial \xi_j} \right) f(\xi), \tag{16}$$

so that the operator (16) should take solutions of equation (6) to solutions, i.e.:

$$\Omega_{X^t}[\delta X^t] U_t(X^t \leftarrow X^0) = U_t(X^t \leftarrow X^0) \Omega_{X^0}[\delta X^0]. \tag{17}$$

Thus, the introduced 1-forms $\omega$ (number-valued) and $\Omega$ (operator-valued) enter to important relations (14) and (17), i.e. they are conserved under time evolution. Here $\delta X^t$ is any solution of variation system for equations (4), (5), i.e.

$$u_t(X^0 + \delta X^0) \simeq X^t + \delta X^t.$$

Another important property is

$$[\Omega_X[\delta X_1]; \Omega_X[\delta X_2]] = i d\omega_X(\delta X_1, \delta X_2).$$

The superposition state (7) of the form

$$\psi(x) = \mathrm{const} \int d\alpha K^h_{X(\alpha)} f(\alpha) \tag{18}$$

may be viewed as a $k$-dimensional surface on the semiclassical bundle $\{(X(\alpha), f(\alpha))\}$; the Maslov isotropic condition (8) and inner product formula (9) can be rewritten as

$$\omega_X \left[ \frac{\partial X}{\partial \alpha_a} \right] = 0, \tag{19}$$

$$(\psi, \psi) \simeq \int d\alpha \left( f(\alpha), \prod_a 2\pi\delta \left( \Omega_X \left[ \frac{\partial X}{\partial \alpha_a} \right] \right) f(\alpha) \right). \tag{20}$$

The comparison of geometric properties of classical, quantum and semiclassical mechanics is presented in Table 2.

**Table 2.** Classical, quantum and semiclassical mechanics.

| | Classical mechanics | Quantum mechanics | Semiclassical mechanics |
|---|---|---|---|
| State space | Symplectic manifold | Hilbert space | Semiclassical bundle: base – symplectic manifold (classical states); fibres – Hilbert spaces (quantum states in classical background) |
| Geometric structures | Symplectic 2-form | Inner product | 1-form on base ("action"); operator-valued 1-form on base (values are operators in fibre); inner products in fibres |
| Symmetry transformations | Lie group acts on manifold | Lie group is represented by unitary operators | Lie group is represented by automorphisms of the bundle |

## 5 Semiclassical symmetries and their infinitesimal properties

Symmetry group $G$ should act on the semiclassical bundle as a group of automorphisms. This means that for each $g \in G$ classical symmetry transformation $u_g : \mathcal{X} \to \mathcal{X}$ is specified; the properties

$$u_e = 1, \qquad u_{g_1 g_2} = u_{g_1} u_{g_2} \tag{21}$$

should be specified. Unitary operators $U_g(u_g X \leftarrow X) : \mathcal{F}_X \to \mathcal{F}_{u_g X}$ such that

$$U_{g_1 g_2}(u_{g_1 g_2} X \leftarrow X) = U_{g_1}(u_{g_1 g_2} X \leftarrow u_{g_2} X) U_{g_2}(u_{g_2} X \leftarrow X) \tag{22}$$

are also specified. The geometric structures (1-forms $\omega$ and $\Omega$) should conserve under symmetry transformations. Therefore, if

$$u_g(X + \delta X_0) = u_g X + \delta X_g + \cdots,$$

one should have

$$\omega_{u_g X}[\delta X_g] = \omega_X[\delta X_0], \tag{23}$$

$$\Omega_{u_g X}[\delta X_g] U_g(u_g X \leftarrow X) = U_g(u_g X \leftarrow X) \Omega_X[\delta X_0]. \tag{24}$$

Properties (23) and (24) provide conservation of Maslov isotropic condition (19) and inner product (20) for superposition states (18).

Consider now the infinitesimal properties of the semiclassical group transformations [8]. Let the semiclassical bundle be trivial (i.e. $\mathcal{F}_X = \mathcal{F}$. For $g(t) = \exp[t\delta g]$, transformations $u_{g(t)}$ are constructed similarly to the classical theory (Table 1): $u_{g(t)}$ takes the initial condition for the equation

$$\frac{dX(t)}{dt} = \nabla[\delta g](X(t)), \qquad X(t) \in \mathcal{X} \tag{25}$$

to the solution at time $t$. Here $\nabla[\delta g](X)$ is a Hamiltonian vector field. The unitary operator $U_g(u_g X \leftarrow X)$ takes the initial condition for the equation

$$i\frac{df(t)}{dt} = H[\delta g|X(t)]f(t), \qquad f(t) \in \mathcal{F} \tag{26}$$

to the solution at time $t$:

$$U_{g(t)}(u_{g(t)}X \leftarrow X) : f(0) \mapsto f(t).$$

Here $H[\delta g|X]$ is a self-adjoint $X$-dependent operator in $\mathcal{F}$.

The infinitesimal analog of the group property (22) is

$$[i\nabla[\delta g_1] - H[\delta g_1|X]; i\nabla[\delta g_2] - H[\delta g_2|X]] = i(i\nabla[\delta g_1;\delta g_2] - H[[\delta g_1, \delta g_2]|X). \tag{27}$$

Invariance of 1-forms implies that

$$\nabla[\delta g]\omega = 0, \tag{28}$$
$$(\nabla[\delta g]\Omega)_X[\delta X] = i[\Omega_X[\delta X]; H[\delta g|X]]. \tag{29}$$

Algebraic property (27) can be interpreted in terms of operators acting in the space of sections of the semiclassical bundle. Namely, let $\Psi$ be a section

$$\Psi = \{\Psi_Y \in \mathcal{F}_Y, Y \in \mathcal{X}\}.$$

Set $\check{U}_g : \Psi \mapsto \check{U}_g\Psi$, with

$$(\check{U}_g\Psi)_Y = U_g(Y \leftarrow u_{g^{-1}}Y)\Psi_{u_{g^{-1}}Y}. \tag{30}$$

Then the group property (22) is simplified:

$$\check{U}_{g_1g_2} = \check{U}_{g_1}\check{U}_{g_2}, \tag{31}$$

so that for $g(t) = \exp[t\delta g]$ one has $\check{U}_g = \exp[-it\check{H}[\delta g]$, with

$$\check{H}[\delta g] = H(\delta g|X) - i\nabla[\delta g].$$

The property (27) reads

$$[\check{H}[\delta g_1]; \check{H}[\delta g_2]]\Psi = -i\check{H}[\delta g_1;\delta g_2]\Psi. \tag{32}$$

The mathematical formulations of the results can be found in [9, 10].

# 6 Further developments and applications

The obtained results can be applied to quantum field theories. The proof of Poincaré invariance of semiclassical scalar field theory is presented in [11].

One can investigate the semiclassical theory [12, 13] of gauge theories [14, 2]. For such constrained systems, the classical extended phase space is not flat ($\mathcal{X} \neq \mathbb{R}^{2n+1}$). It is an $m$-dimensional surface

$$\Lambda_a(X) = 0, \qquad a = \overline{1, m}. \tag{33}$$

Here the constraints $\Lambda_a$ obey the relations

$$\{\Lambda_a; \Lambda_b\} = 0$$

on the constraint surface (33).

Moreover, there is an equivalence relation on the classical phase space. The functions $\Lambda_a$ are generators of classical gauge transformations. The $m$-dimensional gauge group $\mathcal{L}$ acts on the phase space

$$\alpha \in \mathcal{L} \mapsto \lambda_\alpha : \mathcal{X} \to \mathcal{X},$$
$$\lambda_e = 1, \qquad \lambda_{\alpha_1\alpha_2} = \lambda_{\alpha_1}\lambda_{\alpha_2},$$

and classical states $X \sim \lambda_\alpha X$ are set to be equivalent.

In the semiclassical theory, the semiclassical bundle is nontrivial: the inner products in $\mathcal{F}_X$ are different for different $X$. The gauge group acts on the semiclassical bundle. The unitary operators

$$V_\alpha(\lambda_\alpha X \leftarrow X) : \mathcal{F}_X \to \mathcal{F}_{\lambda_\alpha X}$$

such that

$$V_{\alpha_1\alpha_2}(\lambda_{\alpha_1\alpha_2} X \leftarrow X) = V_{\alpha_1}(\lambda_{\alpha_1\alpha_2} X \leftarrow \lambda_{\alpha_2} X) V_{\alpha_2}(\lambda_{\alpha_2} X \leftarrow X)$$

are given. The semiclassical states

$$(X, f) \sim (\lambda_\alpha X, V_\alpha(\lambda_\alpha X \leftarrow X) f) \tag{34}$$

are set to be equivalent.

Introduce a notion of a symmetry group for gauge theories. For $g \in G$, the transformations

$$u_g : \mathcal{X} \to \mathcal{X}; \qquad U_g(u_g X \leftarrow X) : \mathcal{F}_X \to \mathcal{F}_{u_g X}$$

should be given. Gauge equivalent states (34) should be taken to gauge equivalent, i.e.

$$(X_1, f_1) \sim (X_2, f_2)$$

implies

$$(u_g X_1, U_g(u_g X \leftarrow X) f_1) \sim (u_g X_2, U_g(u_g X \leftarrow X) f_2). \tag{35}$$

Requirements (21) and (22) are too strong for gauge theories. One should impose a weaker condition

$$(u_{g_1g_2} X, U_{g_1g_2}(u_{g_1g_2} X \leftarrow X) f) \sim (u_{g_1} u_{g_2} X, U_{g_1}(u_{g_1} u_{g_2} X \leftarrow u_{g_2} X) U_{g_2}(u_{g_2} X \leftarrow X) f). \tag{36}$$

To investigate infinitesimal properties, consider the gauge invariant sections of the semiclassical bundle, which satisfies the condition

$$V_\alpha(\lambda_\alpha Y \leftarrow Y)\Psi_Y = \Psi_{\lambda_\alpha Y}.$$

Introduce the operator $\check{U}_g$ with the help of equation (30). The first requirement (35) means that gauge invariant sections are taken to gauge invariant. The property (36) means that relation (31) is satisfied. Then the infinitesimal property (32) is obeyed for gauge-invariant sections $\Psi$.

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