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# On the BBGKY Hierarchy Solutions for Many-Particle Systems with Different Symmetry Properties

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We construct a new representation to solutions of the initial value problem of the BBGKY hierarchy of equations. Such representation of solutions enable us to describe the cluster nature of evolution of infinite particle systems with various symmetry properties in detail. Convergence of the constructed expansions is investigated in suitable functional spaces.

## 1 Introduction

Various symmetry properties of many-particle systems are generated by the indistinguishability property of identical particles. Classical identical particles are described by observables and states that are symmetric functions with respect to permutations of their arguments (the phase space coordinates of every particle) [1, 2]. In the quantum case we have additional symmetry properties related to the nature of identical particles (Fermi and Bose particles). Classical many-particle systems can also consist of distinguishable particles. In this case many-particle systems are described by observables and states that are non-symmetric functions of their arguments (non-symmetrical particle systems).

The goal of this paper is to analyze the structure of expansions for the BBGKY hierarchy solutions, which depends on the symmetry properties of many-particle systems. We construct a new representation to solutions of the initial value problem of the BBGKY hierarchy in the form of an expansion over particle clusters whose evolution is governed by the cumulant (semi-invariant) of the evolution operator of the corresponding particle cluster for symmetric and non-symmetric systems.

## 2 Cluster expansions of evolution operators of symmetrical particle systems

We consider the initial value problem of the BBGKY hierarchy of equations for a classical system of identical particles [1,2]. If $F(0) = (1, F_1(0, x_1), \ldots, F_s(0, x_1, \ldots, x_s), \ldots)$ is a sequence of initial $s$-particle distribution functions $F_s(0, x_1, \ldots, x_s)$ symmetric in $x_i \equiv (q_i, p_i) \in \mathbb{R}^\nu \times \mathbb{R}^\nu$, $\nu \geq 1$, then a solution $F(t) = (1, F_1(t, x_1), \ldots, F_s(t, x_1, \ldots, x_s), \ldots)$ of the Cauchy problem for the BBGKY hierarchy is represented as the expansion

$$F_s(t, x_1, \ldots, x_s) = \sum_{n=0}^{\infty} \frac{1}{n!} \int_{(\mathbb{R}^\nu \times \mathbb{R}^\nu)^n} dx_{s+1} \cdots dx_{s+n}\, \mathfrak{A}_{(n)}(t, x_1, \ldots, x_s; x_{s+1}, \ldots, x_{s+n}) \times F_{s+n}(0, x_1, \ldots, x_{s+n}), \qquad s \geq 1. \tag{1}$$

where the evolution operator $\mathfrak{A}_{(n)}(t)$ is defined as follows. Let $(x_1, \ldots, x_s) \equiv Y$, $(Y, x_{s+1}, \ldots, x_{s+n}) \equiv X$, i.e., $(x_{s+1}, \ldots, x_{s+n}) = X \backslash Y$, and let $|X| = |Y| + |X \backslash Y| = s + n$ denote the number of elements of the set $X$. Then we have ($|X\backslash Y| \geq 0$)

$$\mathfrak{A}_{(|X\backslash Y|)}(t, Y, X\backslash Y) = \sum_{P:\{Y, X\backslash Y\}=\bigcup_i X_i} (-1)^{|P|-1}(|P|-1)! \prod_{X_i \subset P} S_{|X_i|}(-t, X_i), \tag{2}$$

where $\sum_P$ is the sum over all possible decompositions of the set $\{Y,\, X\backslash Y\}$ into $|P|$ nonempty mutually disjoint subsets $X_i \subset \{Y,\, X\backslash Y\}$, $X_i \cap X_j = \varnothing$, and the set $Y$ completely belongs to one of the subsets $X_i$. In formula (2) the evolution operator $S_{|X|}(-t, X)$ of the Liouville equation describes the dynamics of a system of a finite number $n$ of particles, namely,

$$S_n(-t, x_1, \ldots, x_n)\, f_n(x_1, \ldots, x_n) = f_n\big(X_1(-t, x_1, \ldots, x_n), \ldots, X_n(-t, x_1, \ldots, x_n)\big), \tag{3}$$

where $X_i(-t, x_1, \ldots, x_n)$, $i = 1, \ldots, n$, is a solution of the initial value problem for the Hamilton equations of a system of $n$ particles with initial data $X_i(0, x_1, \ldots, x_n) = x_i$, $i = 1, \ldots, n$, ($S_n(0) = I$ is the identity operator). Operator (3) is defined e.g. in the space of integrable functions $f_n \in L^1(\mathbb{R}^{\nu n} \times \mathbb{R}^{\nu n}) \equiv L^1_n$ [1], in particular, $\|S_n(-t)\|_{L^1_n} = 1$.

The simplest examples of evolution operators $\mathfrak{A}_{(n)}(t)$ (2) have the form

$$\begin{aligned}
&\mathfrak{A}_{(0)}(t, Y) = S_s(-t, Y),\\
&\mathfrak{A}_{(1)}(t, Y, x_{s+1}) = S_{s+1}(-t, Y, x_{s+1}) - S_s(-t, Y)S_1(-t, x_{s+1}),\\
&\mathfrak{A}_{(2)}(t, Y, x_{s+1}, x_{s+2}) = S_{s+2}(-t, Y, x_{s+1}, x_{s+2})\\
&\qquad - S_{s+1}(-t, Y, x_{s+1})S_1(-t, x_{s+2}) - S_{s+1}(-t, Y, x_{s+2})S_1(-t, x_{s+1})\\
&\qquad - S_s(-t, Y)S_2(-t, x_{s+1}, x_{s+2}) + 2!S_s(-t, Y)S_1(-t, x_{s+1})S_1(-t, x_{s+2}).
\end{aligned}$$

Evolution operators (2) are solutions of the following recursion relations ($|X\backslash Y| \geq 0$)

$$S_{|X|}(-t, Y,\, X\backslash Y) = \sum_{P:\{Y,\, X\backslash Y\}=\bigcup_i X_i} \prod_{X_i \subset P} \mathfrak{A}_{(|X_i|-1)}(t, X_i), \tag{4}$$

where $\sum_P$ is the sum as above in formula (2).

We note that recursion relations (4) are typical cluster expansions [3] for the evolution operator $S_{|X|}(-t, Y, X\backslash Y)$ defined by (3). Thus, the operators $\mathfrak{A}_{(|X\backslash Y|)}(t, Y, X\backslash Y)$ (2) have the meaning of the cumulants (semi-invariants) of the operator $S_{|X|}(-t, Y, X\backslash Y)$ describing the evolution of a system of a finite number $|X|$ of particles, i.e., they describe what noninteracting clusters of particles may form a system of the corresponding number of particles in the process of evolution, provided that a cluster of $|Y|$ particles evolves as a single cluster.

The structure of the cluster expansions (4) can be represented in a more explicit and compact form. To this end we consider the set of sequences $\Psi = \big(\Psi_0, \Psi_1(x_1), \ldots, \Psi_n(x_1, \ldots, x_n), \ldots\big)$ of operators $\Psi_n$ of type (3) ($\Psi_0$ is an operator that multiplies a function by an arbitrary number). In this set we introduce the tensor $*$-product

$$(\Psi_1 * \Psi_2)_{|X|}(X) = \sum_{Y \subset X} (\Psi_1)_{|Y|}(Y)\, (\Psi_2)_{|X\backslash Y|}(X\backslash Y),$$

where $\sum_{Y\subset X}$is the sum over all subsets $Y$ of the set $X \equiv (x_1, \ldots, x_n)$.

As a result, expression (2) can be rewritten in the form

$$\mathfrak{A}(t) = \mathbb{L}\mathrm{n}_*\big(\mathbf{1} + S(-t)\big), \tag{5}$$

where $\mathfrak{A}(t) = (0, (\mathfrak{A}(t))_1(Y), (\mathfrak{A}(t))_2(Y, x_{s+1}), \ldots)$ and $(\mathfrak{A}(t))_{1+n}(Y, x_{s+1}, \ldots, x_{s+n}) = \mathfrak{A}_{(n)}(t, Y, X\backslash Y) \equiv \mathfrak{A}_{|Y|+n}(t, Y, X\backslash Y)$. A mapping $\mathbb{L}\mathrm{n}_*$ is defined: $\mathbb{L}\mathrm{n}_*(\mathbf{1} + \Psi) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \underbrace{\Psi * \cdots * \Psi}_{n}$, ($\Psi = (0, \Psi_1, \ldots, \Psi_n, \ldots)$, and $\mathbf{1} = (1, 0, 0, \ldots)$ is the unit sequence).

We also remark that the cluster expansions (4) have the form

$$\mathbf{1} + S(-t) = \mathbb{E}\mathrm{xp}_*\mathfrak{A}(t),$$

where $\mathbb{E}\mathrm{xp}_*$ is defined as the $*$-exponential mapping: $\mathbb{E}\mathrm{xp}_*\Psi = \mathbf{1} + \sum\limits_{n=1}^{\infty} \frac{1}{n!} \underbrace{\Psi * \cdots * \Psi}_{n}$.

We note that the connections between different representations of the BBGKY hierarchy solutions are considered in [1,3]. For the first time several first terms of the expansion (1), (2) for the one-particle distribution function were determined in [4,5].

# 3 Cumulant representation of BBGKY hierarchy solutions for symmetrical particle systems

We consider the problem of the convergence of the expansion (1), (2) in the space of sequences of integrable functions and prove the solution existence theorem for the initial data from this space.

Let $L^1_\alpha = \bigoplus_{n=0}^{\infty} \alpha^n L^1_n$ be the Banach space of sequences $f = (f_0, f_1(x_1), \ldots, f_n(x_1, \ldots, x_n), \ldots)$ of symmetric integrable functions $f_n(x_1, \ldots, x_n)$ defined on the phase space $\mathbb{R}^{\nu n} \times \mathbb{R}^{\nu n}$ with the norm

$$\|f\| = \sum_{n=0}^{\infty} \alpha^n \|f_n\|_{L^1_n} = \sum_{n=0}^{\infty} \alpha^n \int_{(\mathbb{R}^\nu \times \mathbb{R}^\nu)^n} dx_1 \cdots dx_n \, |f_n(x_1, \ldots, x_n)|,$$

where $\alpha > 1$ is a number; $L^1_{\alpha,0} \subset L^1_\alpha$ is the subspace of finite sequences of continuously differentiable functions with compact supports.

Since on the sequences of integrable functions $f \in L^1_\alpha$ the annihilation operator [1,2]

$$\big(\mathfrak{a} f\big)_n(x_1, \ldots, x_n) = \int_{\mathbb{R}^\nu \times \mathbb{R}^\nu} dx_{n+1} \, f_{n+1}(x_1, \ldots, x_n, x_{n+1}), \tag{6}$$

is defined, in view of (5) and (6), expression (1), (2) takes the following form in the space $L^1_\alpha$

$$F(t) = e^{\mathfrak{a}}\mathfrak{A}(t)F(0) = e^{\mathfrak{a}}\mathbb{L}\mathrm{n}_*\big(\mathbf{1} + S(-t)\big)F(0). \tag{7}$$

If $F(0) \in L^1_{s+n}$, then the following estimate is true:

$$\|\mathfrak{A}_{(n)}(t)F_{s+n}(0)\|_{L^1_{s+n}} \leq n! e^{n+2} \|F_{s+n}(0)\|_{L^1_{s+n}}. \tag{8}$$

By virtue of inequality (8), the functions defined by (1) (or (7)) satisfy the following estimate for $\alpha > e$:

$$\|F(t)\|_{L^1_\alpha} \leq c_\alpha \|F(0)\|_{L^1_\alpha}, \tag{9}$$

where $c_\alpha = e^2 \left(1 - \frac{e}{\alpha}\right)^{-1}$ is a constant.

Note that the parameter $\alpha$ can be interpreted as a quantity inversely proportional to the density of the system (the average number of particles in a unit volume).

Thus, according to estimate (9) the following existence theorem is true:

**Theorem 1.** *If $F(0) \in L^1_\alpha$ is a sequence of nonnegative functions, then for $\alpha > e$, and $t \in \mathbb{R}^1$, there exists a unique solution to the initial value problem for the BBGKY hierarchy, namely the sequence $F(t) \in L^1_\alpha$ of nonnegative functions $F_s(t)$ defined by*

$$F_{|Y|}(t, Y) = \sum_{n=0}^{\infty} \frac{1}{n!} \int d(X \backslash Y) \sum_{P: \{Y, X \backslash Y\} \bigcup_i X_i} (-1)^{|P|-1} (|P| - 1)! \\ \times \prod_{X_i \subset P} S_{|X_i|}(-t, X_i) F_{|X|}(0, Y, X \backslash Y), \tag{10}$$

*where we have the same notations as for* (2)*. This solution is a strong solution for* $F(0) \in L^1_{\alpha,0}$ *and a weak one for arbitrary initial data.*

If we consider initial data $F(0)$ from the space $L^\infty_{\xi,\beta}$ of sequences of continuous symmetric functions bounded with respect to the configuration variables [1,2] the problem of the divergence of configuration integrals in each term of series (1), (2) arises. We note that the cumulant structure of expansion (1) allows one to prove the possibility of eliminating this divergence.

## 4 Cluster expansions of evolution operators of non-symmetrical particle systems

We consider a one-dimensional system of identical particles interacting with their nearest neighbours via the hard-core pair potential $\Phi$. For the configurations ($q_i \in \mathbb{R}^1$ is the position of the $i$ particle center) of such a system the following inequalities must be satisfied: $\sigma + q_i \le q_{i+1}$, where $\sigma$ is the length of a particle, and the natural way to number the particles is to number by means of the integers from the set $\mathbb{Z}\backslash\{0\}$. The Hamiltonian of the $n = n_1 + n_2$ particle system

$$H_n = \sum_{i\in(-n_2,\ldots,-1,1,\ldots,n_1)} \frac{p_i^2}{2} + \sum_{(i,i+1)\in\big((-n_2,-n_2+1),\ldots,(n_1-1,n_1)\big)} \Phi(q_i - q_{i+1})$$

is a function non-symmetrical [6,7] with respect to permutations of the arguments $x_i \equiv (q_i, p_i) \in \mathbb{R}^1 \times \mathbb{R}^1$.

If $F(0) = \big\{F_s(0, x_{-s_2}, \ldots, x_{s_1})\big\}_{s=s_1+s_2\ge 0}$ is a sequence of initial $s$-particle distribution functions $F_s(0, x_{-s_2}, \ldots, x_{-1}, x_1, \ldots, x_{s_1})$, $F_0 = 1$, then a solution of the Cauchy problem for the BBGKY hierarchy $F(t) = \big\{F_s(t, x_{-s_2}, \ldots, x_{s_1})\big\}_{s=s_1+s_2\ge 0}$ is represented as the expansion

$$F_s(t, x_{-s_2}, \ldots, x_{s_1}) = \sum_{n=0}^{\infty} \sum_{\substack{n = n_1 + n_2 \\ n_1,\ n_2 \ge 0}} \int_{(\mathbb{R}^1\times\mathbb{R}^1)^{n_1+n_2}} dx_{-(n_2+s_2)} \cdots dx_{-(s_2+1)} \\ \times dx_{s_1+1} \cdots dx_{s_1+n_1} \big(\mathfrak{A}_{(n_2,n_1)}(t)\, F_{s+n}(0)\big)(x_{-(n_2+s_2)}, \ldots, x_{s_1+n_1}), \tag{11}$$

where the evolution operator $\mathfrak{A}_{(n_2,n_1)}(t)$ is defined in the following way. Let $(x_{-s_2}, \ldots, x_{s_1}) \equiv Y$, $(x_{-(n_2+s_2)}, \ldots, x_{s_1+n_1}) \equiv X$. The sets $X$ and $Y$ are partially ordered sets, because $\sigma + q_i \le q_{i+1}$. If the subset $Y$ of the set $X$ is treated as one element similar to $(x_{-(n_2+s_2)}, \ldots, x_{-(s_2+1)}$, $x_{s_1+1}, \ldots, x_{s_1+n_1})$, then for such a partially ordered set we use the notation $X_Y$. Symbol $|Y| = s = s_1 + s_2$ denotes the number of elements of the set $Y$ and, thus, $|X_Y| = n_1 + n_2 + 1$. Then we have

$$\mathfrak{A}_{(n_2,n_1)}(t, X_Y) = \sum_{P:X_Y=\bigcup_i X_i} (-1)^{|P|-1} \prod_{X_i\subset P} S_{|X_i|}(-t, X_i), \tag{12}$$

where $n_1 + n_2 = n \ge 0$, $\sum_P$ is the sum over all ordered decompositions of the partially ordered set $X_Y$ into $|P|$ nonempty partially ordered subsets $X_i \subset X_Y$, which are mutually disjoint $X_i \bigcap X_j = \varnothing$, and the set $Y$ completely belongs to one of the subsets $X_i$. As above, in formula (12) the evolution operator $S_{|X|}(-t, X)$ describes the dynamics of a system of a finite number $n = n_1 + n_2$ of particles [2,6].

The evolution operators (12) are solutions of the following recursion relations ($|X_Y| - 1 \ge 0$)

$$S_{|X|}(-t, X) = \sum_{P:X_Y=\bigcup_i X_i} \prod_{X_i\subset P} \mathfrak{A}_{(i_2,\ i_1)}(t, X_i), \tag{13}$$

where $i_1 + i_2 = i = |X_i| - 1 \geq 0$, and $\sum_P$ is the sum given above in formula (12). The recursion relations (13) are cluster expansions for the evolution operator of non-symmetrical particle systems. We note that the structure of cluster expansion (13) is essentially different from the structure of corresponding expansions (4) for the symmetrical systems.

As above, the structure of the cluster expansions (13) can be represented in a more explicit and compact form. In the set of double sequences $\Psi = \{\Psi_{n_1+n_2}(x_{-n_2}, \ldots, x_{n_1})\}_{n_1+n_1=n\geq 0}$ of operators $\Psi_{n_1+n_2}$ we introduce the following tensor $\star$-product

$$(\Psi_1 \star \Psi_2)_{|X|}(X) = \sum_{Y\subset X} (\Psi_1)_{|Y|}(Y)\, (\Psi_2)_{|X\backslash Y|}(X\backslash Y),$$

where $\sum_{Y\subset X}$ is the sum over all partially ordered subsets $Y$ of the partially ordered set $X \equiv (x_{-n_2}, \ldots, x_{n_1})$.

Then expression (12) for the cumulants of non-symmetrical systems can be rewritten in the form

$$\mathfrak{A}(t) = \mathbf{1} - (\mathbf{1} + \mathbf{S}(-\mathbf{t}))^{-\mathbf{1}_\star}, \tag{14}$$

where $\mathfrak{A}(t) = \big(0, (\mathfrak{A}(t))_1(Y), \ldots, (\mathfrak{A}(t))_{1+n_1+n_2}(X_Y), \ldots\big)$ and $(\mathfrak{A}(t))_{1+n_1+n_2}(x_{-(n_2+s_2)}, \ldots, x_{s_1+n_1}, Y, x_{s_1+1}, \ldots, x_{s_1+n_1}) \equiv (\mathfrak{A}(t))_{1+n}(X_Y) = \mathfrak{A}_{(n_2,n_1)}(t, X_Y)$. A mapping $\mathbf{1} - (\mathbf{1} + \cdot)^{-1_\star}$ is defined the by formula: $\mathbf{1} - (\mathbf{1} + \Psi)^{-1_\star} = \sum_{n=1}^{\infty} (-1)^{n-1} \underbrace{\Psi \star \cdots \star \Psi}_{n}$ ($\Psi \equiv (0, \Psi_1, \ldots, \Psi_{1+n_1+n_2}, \ldots)$ and $\mathbf{1} = (1, 0, 0, \ldots)$ is the unit sequence).

The cluster expansions (13) for the evolution operator of non-symmetrical particle systems have the form

$$\mathbf{1} + S(-t) = \big(\mathbf{1} - \mathfrak{A}(t)\big)^{-1_\star},$$

where $(\mathbf{1} - \cdot)^{-1_\star}$ is defined as the $\star$-resolvent: $\big(\mathbf{1} - \Psi\big)^{-1_\star} = \mathbf{1} + \sum_{n=1}^{\infty} \underbrace{\Psi \star \cdots \star \Psi}_{n}$.

## 5 Cumulant representation of BBGKY hierarchy solutions for non-symmetrical particle systems

We consider the problem of the convergence of expansion (11), (12) in the space of sequences of integrable functions. Let $L^1_\alpha = \sum_{n=0}^{\infty} \bigoplus_{\substack{n = n_1+n_2 \\ n_1,\ n_2 \geq 0}} \alpha^{n_1+n_2} L^1_{n_1+n_2}$ be the Banach space of double sequences $f = \{f_n(x_{-n_2}, \ldots, x_{n_1})\}_{n=n_1+n_2\geq 0}$ integrable functions $f_n(x_{-n_2}, \ldots, x_{n_1})$ defined on the phase space $\mathbb{R}^n \times (\mathbb{R}^n \backslash W_n)$ [2, 7] with the norm

$$\|f\| = \sum_{n=0}^{\infty} \sum_{\substack{n = n_1+n_2 \\ n_1,\ n_2 \geq 0}} \alpha^{n_1+n_2} \int_{(\mathbb{R}^1\times\mathbb{R}^1)^{n_1+n_2}} dx_{-n_2} \cdots dx_{n_1}\, |f_{n_1+n_2}(x_{-n_2}, \ldots, x_{n_1})|,$$

where $\alpha > 1$ is a number; $L^1_{\alpha,0} \subset L^1_\alpha$ is the subspace of finite sequences of continuously differentiable functions with compact supports.

Since on the sequences of integrable functions $f \in L^1_\alpha$ the operators

$$\big(\mathfrak{a}_{(+)} f\big)_n(x_{-n_2}, \ldots, x_{n_1}) = \int_{\mathbb{R}^1\times\mathbb{R}^1} dx_{n_1+1}\, f_{n+1}(x_{-n_2}, \ldots, x_{n_1}, x_{n_1+1}),$$

$$\big(\mathfrak{a}_{(-)} f\big)_n(x_{-n_2}, \ldots, x_{n_1}) = \int_{\mathbb{R}^1\times\mathbb{R}^1} dx_{-(n_2+1)}\, f_{n+1}(x_{-(n_2+1)}, x_{-n_2}, \ldots, x_{n_1})$$

are defined, in view of (14) expansion (11) takes the following form in the space $L^1_\alpha$

$$F(t) = \big(\mathbf{1} - \mathfrak{a}_{(+)}\big)^{-1}\big(\mathbf{1} - \mathfrak{a}_{(-)}\big)^{-1}\mathfrak{A}(t)F(0)$$
$$= \big(\mathbf{1} - \mathfrak{a}_{(+)}\big)^{-1}\big(\mathbf{1} - \mathfrak{a}_{(-)}\big)^{-1}\big(\mathbf{1} - (\mathbf{1} + S(-t))^{-1_\star}\big)F(0).$$

If $F(0) \in L^1_{s+n}$, then the estimates are true:

$$\|\mathfrak{A}_{(n_2,n_1)}(t)\, F_{s+n}(0)\|_{L^1_{s+n}} \leq 2^{\ n_1+n_2}\, \|F_{s+n}(0)\|_{L^1_{s+n}}.$$

By virtue of this inequality, the functions $F(t)$ defined by (11), (12) satisfy the following estimate for $\alpha > 2$,

$$\|F(t)\|_{L^1_\alpha} \leq c^2_\alpha \|F(0)\|_{L^1_\alpha}, \tag{15}$$

where $c_\alpha = \big(1 - \frac{2}{\alpha}\big)^{-1}$ is a constant.

Thus, according to (15) the following existence theorem holds:

**Theorem 2.** *If $F(0) \in L^1_\alpha$ is a sequence of nonnegative functions, then for $\alpha > 2$, and $t \in \mathbb{R}^1$, there exists a unique solution of the initial value problem for the BBGKY hierarchy, namely the sequence $F(t) \in L^1_\alpha$ of nonnegative functions $F_{s_1+s_2}(t)$ defined by*

$$F_{|Y|}(t, Y) = \sum_{n=0}^{\infty} \sum_{\substack{n = n_1 + n_2 \\ n_1,\ n_2 \geq 0}} \int_{(\mathbb{R}^1\times\mathbb{R}^1)^{n_1+n_2}} d(X\backslash Y) \sum_{P: X_Y = \bigcup_i X_i} (-1)^{|P|-1} \times \prod_{X_i \subset P} S_{|X_i|}(-t, X_i) F_{|X|}(0, X),$$

*where we have the same notations as for relations* (12)*. This solution is a strong solution for $F(0) \in L^1_{\alpha,0}$ and a weak one for arbitrary initial data in the space $L^1_\alpha$.*

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