

Splitting the Kemmer–Duffin–Petiau Equations

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We study internal structure of the Kemmer–Duffin–Petiau equations for spin-0 and spin-1 mesons. We demonstrate, that the Kemmer–Duffin–Petiau equations can be splitted into constituent equations, describing particles with definite mass and broken Lorentz symmetry. We also show that solutions of the three-component constituent equations fulfill the Dirac equation.

1 Introduction

In recent years there has been a renewed interest in the Kemmer–Duffin–Petiau (KDP) theory describing spin-0 and spin-1 mesons [1] due to discovery of a new conserved four-vector current with positive zeroth component [2], which can be thus interpreted as a probability density. A progress was also made in demonstrating equivalence of the KDP and the Klein–Gordon equations, especially when interactions are taken into account, c.f. [3] and references therein. The KDP equations has been also studied in the context of electromagnetic interactions [4–6], parasupersymmetric quantum mechanics [4], EPR type nonlocality [7], and Riemann–Cartan space-time [8].

It is well known that the KDP equations contain redundant components – only $2(2s + 1)$ components are needed to describe free spin- s particles with nonzero rest masses [9] while spin-0 and spin-1 KDP equations contain 5 and 10 components, respectively. The presence of redundant components in KDP equations leads for some interactions to nonphysical effects such as superluminal velocities [10,9]. It is possible, however, to obtain physically acceptable equations for arbitrary spin removing redundant components with use of additional covariant condition [9]. On the other hand, presence of redundant components suggests that the KDP equations possess internal structure. The aim of the present paper is to investigate this inner structure. We shall describe a systematic procedure of splitting (five-component) spin-0 and (ten-component) spin-1 KDP equations by means of the spinor calculus into pairs of constituents equations with smaller numbers of components, such that solutions of the latter equations fulfill the initial KDP equations. Since mesons are spin-0 and spin-1 quark-antiquark bound states it is tempting to recognize the resulting equations as quark equations. Indeed, we shall show that solutions of constituent equations fulfill the Dirac equation.

The paper is organized as follows. In Section 2 the Kemmer–Duffin–Petiau equations for spin 0 and spin 1 are described, and necessary definitions and conventions are given. In Section 3 splitting of the KDP equations into three-component constituent equations is achieved for $s = 0$ (obtaining in a new way our previous result [11]). Main results are described in the last two Sections. In Section 4 we interpret the constituent equations finding direct relation with the Dirac equation. In the last Section we outline the procedure of splitting the KDP equations for $s = 1$, and discuss our results in the light of several current problems of quark theory.

2 Kemmer–Duffin–Petiau equations

In what follows tensor indices are denoted with Greek letters, $\mu = 0, 1, 2, 3$. We shall use the following convention for the metric tensor: $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ and we shall always sum over repeated indices. Four-momentum operators are defined in natural units ($c = 1, \hbar = 1$) as $p^\mu = i \frac{\partial}{\partial x_\mu}$.

The KDP equations for spin 0 and 1 are written as:

$$\beta_\mu p^\mu \Psi = m \Psi, \quad (1)$$

with 5×5 and 10×10 matrices β^μ , respectively, which fulfill the following commutation relations [1]:

$$\beta^\lambda \beta^\mu \beta^\nu + \beta^\nu \beta^\mu \beta^\lambda = g^{\lambda\mu} \beta^\nu + g^{\nu\mu} \beta^\lambda. \quad (2)$$

In the case of 5×5 (spin-0) representation of β^μ matrices equation (1) is equivalent to the following set of equations:

$$\begin{aligned} p^\mu \psi &= m \psi^\mu, \\ p_\nu \psi^\nu &= m \psi, \end{aligned} \quad (3)$$

$\mu, \nu = 0, 1, 2, 3$, if we define Ψ in (1) as:

$$\Psi = (\psi^\mu, \psi)^T = (\psi^0, \psi^1, \psi^2, \psi^3, \psi)^T, \quad (4)$$

where $()^T$ denotes transposition of a matrix. Let us note that equation (3) can be obtained by factorizing second-order derivatives in the Klein–Gordon equation $p_\mu p^\mu \psi = m^2 \psi$.

In the case of 10×10 (spin-1) representation of matrices β^μ equation (1) reduces to:

$$\begin{aligned} p^\mu \psi^\nu - p^\nu \psi^\mu &= m \psi^{\mu\nu}, \\ p_\mu \psi^{\mu\nu} &= m \psi^\nu, \end{aligned} \quad (5)$$

$\mu, \nu = 0, 1, 2, 3$, with the following definition of Ψ in (1):

$$\Psi = (\psi^{\mu\nu}, \psi^\lambda)^T = (\psi^{01}, \psi^{02}, \psi^{03}, \psi^{23}, \psi^{31}, \psi^{12}, \psi^0, \psi^1, \psi^2, \psi^3)^T, \quad (6)$$

where ψ^λ are real and $\psi^{\mu\nu}$ are purely imaginary (in alternative formulation we have $-\partial^\mu \psi^\nu + \partial^\nu \psi^\mu = m \psi^{\mu\nu}$, $\partial_\mu \psi^{\mu\nu} = m \psi^\nu$, where $\psi^\lambda, \psi^{\mu\nu}$ are real). Because of antisymmetry of $\psi^{\mu\nu}$ we have $p_\nu \psi^\nu = 0$ what implies spin 1 condition. The set of equations (5) was first written by Proca [1].

3 Splitting the spin-0 Kemmer–Duffin–Petiau equations

Equations (3) can be written within spinor formalism as:

$$\begin{aligned} p^{A\dot{B}} \psi &= m \psi^{A\dot{B}}, \\ p_{A\dot{B}} \psi^{A\dot{B}} &= 2m \psi, \end{aligned} \quad (7)$$

$A = 1, 2, \dot{B} = \dot{1}, \dot{2}$, where the spinor components are defined as [1]:

$$p^{A\dot{B}} = (p^0 \sigma^0 + \sigma \cdot \mathbf{p})^{A\dot{B}} = \begin{pmatrix} p^0 + p^3 & p^1 - ip^2 \\ p^1 + ip^2 & p^0 - p^3 \end{pmatrix}, \quad (8)$$

where σ^0 is the 2×2 unit matrix and σ^i , $i = 1, 2, 3$, are the Pauli matrices (analogously, $\psi^{A\dot{B}} = (\psi^0 \sigma^0 + \boldsymbol{\psi} \cdot \boldsymbol{p})^{A\dot{B}}$).

Splitting the last of equations (7), $p_{A\dot{B}} \psi^{A\dot{B}} = p_{1\dot{1}} \psi^{1\dot{1}} + p_{2\dot{1}} \psi^{2\dot{1}} + p_{1\dot{2}} \psi^{1\dot{2}} + p_{2\dot{2}} \psi^{2\dot{2}} = 2m\psi$, we obtain two sets of equations involving components $\psi^{1\dot{1}}$, $\psi^{2\dot{1}}$, ψ and $\psi^{1\dot{2}}$, $\psi^{2\dot{2}}$, ψ , respectively:

$$\begin{aligned} p^{1\dot{1}} \psi &= m\psi^{1\dot{1}}, \\ p^{2\dot{1}} \psi &= m\psi^{2\dot{1}}, \\ p_{1\dot{1}} \psi^{1\dot{1}} + p_{2\dot{1}} \psi^{2\dot{1}} &= m\psi; \end{aligned} \tag{9}$$

$$\begin{aligned} p^{1\dot{2}} \psi &= m\psi^{1\dot{2}}, \\ p^{2\dot{2}} \psi &= m\psi^{2\dot{2}}, \\ p_{1\dot{2}} \psi^{1\dot{2}} + p_{2\dot{2}} \psi^{2\dot{2}} &= m\psi, \end{aligned} \tag{10}$$

each of which describes particle with mass m (we check this substituting e.g. $\psi^{1\dot{1}}$, $\psi^{2\dot{1}}$ or $\psi^{1\dot{2}}$, $\psi^{2\dot{2}}$ into the third equations). The splitting preserving $p_\mu p^\mu \psi = m^2 \psi$ is possible due to spinor identities

$$p_{1\dot{1}} p^{1\dot{1}} + p_{2\dot{1}} p^{2\dot{1}} = p_{1\dot{2}} p^{1\dot{2}} + p_{2\dot{2}} p^{2\dot{2}} = p_\mu p^\mu, \tag{11}$$

which follow directly from (8).

Thus solutions of equations (9), (10) fulfill the KDP equations (7). We described these equations in [11]. From each of equations(9), (10) an identity follows:

$$p^{2\dot{1}} \psi^{1\dot{1}} = p^{1\dot{1}} \psi^{2\dot{1}}, \tag{12a}$$

$$p^{2\dot{2}} \psi^{1\dot{2}} = p^{1\dot{2}} \psi^{2\dot{2}}. \tag{12b}$$

Equations (9), (10) can be written in matrix form:

$$\rho_\mu p^\mu \Phi = m\Phi, \tag{13}$$

where $\Phi = (\psi^{1\dot{1}}, \psi^{2\dot{1}}, \psi)^T$,

$$\begin{aligned} \rho^0 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & \rho^1 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \\ \rho^2 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & -i & 0 \end{pmatrix}, & \rho^3 &= \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \end{aligned} \tag{14}$$

and

$$\tilde{\rho}_\mu p^\mu \tilde{\Phi} = m\tilde{\Phi}, \tag{15}$$

where $\tilde{\Phi} = (\psi^{1\dot{2}}, \psi^{2\dot{2}}, \psi)^T$,

$$\begin{aligned} \tilde{\rho}^0 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, & \tilde{\rho}^1 &= \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \\ \tilde{\rho}^2 &= \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, & \tilde{\rho}^3 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}. \end{aligned} \tag{16}$$

Equations (13), (15) considered together:

$$\begin{pmatrix} \rho_\mu p^\mu & \mathbf{0} \\ \mathbf{0} & \tilde{\rho}_\mu p^\mu \end{pmatrix} \begin{pmatrix} \Phi \\ \tilde{\Phi} \end{pmatrix} = m \begin{pmatrix} \Phi \\ \tilde{\Phi} \end{pmatrix}, \quad (17)$$

are Lorentz covariant since they contain all components of the spinor $\psi^{A\dot{B}}$. Obviously, all solutions of equation (17) satisfy equation (7) but the reverse is not true.

The matrices: ρ^μ , $\tilde{\rho}^\mu$ discussed above, c.f. equations (14), (16), fulfill the Tzou commutation relations [11, 4]

$$\rho^{(\lambda} \rho^\mu \rho^{\nu)} = g^{(\lambda\mu} \rho^{\nu)}, \quad (18)$$

more complicated than (2), where $(\lambda \mu \nu)$ is the symmetrizer. There is, however, no conjugation rule for matrices ρ^μ and $\tilde{\rho}^\mu$, for example there is no such matrix S that $\tilde{\rho}^\mu = S\rho^\mu S^{-1}$. We shall see in the next Section that a conjugation rule (charge conjugation) exists if 3×3 matrices ρ^μ are extended to 4×4 Dirac matrices γ^μ .

4 Subolutions of the Dirac equation

We shall now interpret the constituent equations (9), (10) together with the identities (12a), (12b). Equation (9) and the identity (12a), as well as equation (10) and the identity (12b) can be written in form of the Dirac equations:

$$\begin{pmatrix} 0 & 0 & p^0 + p^3 & p^1 - ip^2 \\ 0 & 0 & p^1 + ip^2 & p^0 - p^3 \\ p^0 - p^3 & -p^1 + ip^2 & 0 & 0 \\ -p^1 - ip^2 & p^0 + p^3 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi^{1\dot{1}} \\ \psi^{2\dot{1}} \\ \chi \\ 0 \end{pmatrix} = m \begin{pmatrix} \psi^{1\dot{1}} \\ \psi^{2\dot{1}} \\ \chi \\ 0 \end{pmatrix}, \quad (19)$$

$$\begin{pmatrix} 0 & 0 & p^0 - p^3 & p^1 + ip^2 \\ 0 & 0 & p^1 - ip^2 & p^0 + p^3 \\ p^0 + p^3 & -p^1 - ip^2 & 0 & 0 \\ -p^1 + ip^2 & p^0 - p^3 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi^{2\dot{2}} \\ \psi^{1\dot{2}} \\ \chi \\ 0 \end{pmatrix} = m \begin{pmatrix} \psi^{2\dot{2}} \\ \psi^{1\dot{2}} \\ \chi \\ 0 \end{pmatrix}, \quad (20)$$

respectively, with one zero component. Equation (19) can be written as $\gamma^\mu p_\mu \Psi = m\Psi$ with spinor representation of the Dirac matrices, $\gamma^0 = \begin{pmatrix} \mathbf{0} & \sigma^0 \\ \sigma^0 & \mathbf{0} \end{pmatrix}$, $\gamma^j = \begin{pmatrix} \mathbf{0} & -\sigma^j \\ \sigma^j & \mathbf{0} \end{pmatrix}$, $j = 1, 2, 3$, $\gamma^5 = \begin{pmatrix} \sigma^0 & \mathbf{0} \\ \mathbf{0} & -\sigma^0 \end{pmatrix}$, $\Psi = (\psi^{1\dot{1}}, \psi^{2\dot{1}}, \chi, 0)^T$. Equation (20) can be analogously written as $(\gamma^0 p^0 - \gamma^1 p^1 + \gamma^2 p^2 + \gamma^3 p^3) \Phi = m\Phi$, $\Phi = (\psi^{2\dot{2}}, \psi^{1\dot{2}}, \chi, 0)^T$.

We shall demonstrate now that equations (19) and (20) are charge conjugated one to another. Complex conjugation of equation (19) yields:

$$(-1) \begin{pmatrix} 0 & 0 & p^0 + p^3 & p^1 + ip^2 \\ 0 & 0 & p^1 - ip^2 & p^0 - p^3 \\ p^0 - p^3 & -p^1 - ip^2 & 0 & 0 \\ -p^1 + ip^2 & p^0 + p^3 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi^{1\dot{1}} \\ \psi^{2\dot{1}} \\ \chi \\ 0 \end{pmatrix}^* = m \begin{pmatrix} \psi^{1\dot{1}} \\ \psi^{2\dot{1}} \\ \chi \\ 0 \end{pmatrix}^*, \quad (21)$$

i.e. $(-1) (\gamma^0 p^0 - \gamma^1 p^1 + \gamma^2 p^2 - \gamma^3 p^3) \Psi^* = m\Psi^*$ where $*$ denotes complex conjugation. Acting from the left with matrix γ^3 on equation (21) we obtain equation $(\gamma^0 p^0 - \gamma^1 p^1 + \gamma^2 p^2 + \gamma^3 p^3) \times \gamma^3 \Psi^* = m\gamma^3 \Psi^*$, which has the same form as equation (20) (the charge conjugation matrix C is

thus defined as $C\gamma^0 \equiv \gamma^3$ [12]). Hence the initial equations (9), (10) are charge conjugated one to another in a sense that they are charge conjugated after extension to the Dirac form.

The observations made above can be given representation independent formulation. Let us notice that the three component equations, for instance (9), (10) as well as the identities (12a), (12b), can be obtained by projecting the Dirac equation with projection operator $P_4 = \text{diag}(1, 1, 1, 0)$. Incidentally, there are other projection operators which lead to analogous three component equations, $P_1 = \text{diag}(0, 1, 1, 1)$, $P_2 = \text{diag}(1, 0, 1, 1)$, $P_3 = \text{diag}(1, 1, 0, 1)$ but we shall need only the operator P_4 .

In general, we can consider subsolutions, of form $P_4\Psi$, of the Dirac equation:

$$\gamma^\mu p_\mu P_4\Psi = mP_4\Psi, \quad (22)$$

which is equivalent to (19) in the case of spinor representation of the Dirac matrices.

Accordingly, acting from the left on (22) with P_4 and $(1 - P_4)$ we obtain two equations:

$$P_4(\gamma^\mu p_\mu)P_4\Psi = mP_4\Psi, \quad (23a)$$

$$(1 - P_4)(\gamma^\mu p_\mu)P_4\Psi = 0. \quad (23b)$$

In the spinor representation of γ^μ matrices equation (23a) is equivalent to (9), while (23b) is equivalent to the identity (12a).

Now the projection operator can be written as $P_4 = \frac{1}{4}(3 + \gamma^5 - \gamma^0\gamma^3 + i\gamma^1\gamma^2)$ (and similar formulae can be given for other projection operators P_1, P_2, P_3), i.e. all equations (22), (23a), (23b) are now given representation independent form. Let us also note that the projection operator P_4 commutes with two generators of Lorentz transformations $\gamma^0\gamma^3$ and $\gamma^1\gamma^2$ (and does not commute with other generators), i.e. is invariant under boosts in x^0x^3 plane and rotations in x^1x^2 plane. Accordingly, the three component equations are covariant with respect to such Lorentz transformations only. Let us note finally that all three component equations describe particles with definite mass and partly undefined spin.

5 Discussion

We shall now approach the problem of splitting the KDP equations for $s = 1$. Equations (5) can be written in spinor form as [1]:

$$\begin{aligned} p_A^{\dot{B}} \zeta_{C\dot{B}} + p_C^{\dot{B}} \zeta_{A\dot{B}} &= 2m\eta_{AC}, \\ p_B^A \zeta_{A\dot{D}} + p_D^A \zeta_{A\dot{B}} &= 2m\chi_{\dot{B}\dot{D}}, \\ p_A^{\dot{C}} \chi_{\dot{B}\dot{C}} + p_B^C \eta_{AC} &= -2m\zeta_{A\dot{B}}. \end{aligned} \quad (24)$$

It is possible to split the spinor form of the KDP equations (24) to get two equations for spinors $\chi_{\dot{B}\dot{D}}$, $\zeta_{A\dot{B}}$ and η_{AC} , $\zeta_{A\dot{B}}$:

$$p_A^{\dot{B}} \zeta_{C\dot{B}} = m\eta_{AC}, \quad \eta_{AC} = \eta_{CA}, \quad p_B^C \eta_{AC} = -m\zeta_{A\dot{B}}, \quad (25)$$

$$p_B^A \zeta_{A\dot{D}} = m\chi_{\dot{B}\dot{D}}, \quad \chi_{\dot{B}\dot{D}} = \chi_{\dot{D}\dot{B}}, \quad p_A^{\dot{D}} \chi_{\dot{B}\dot{D}} = -m\zeta_{A\dot{B}}, \quad (26)$$

respectively. The splitting is possible due to spinor identities:

$$p_B^C p_A^{\dot{B}} = -\delta_A^C p_\mu p^\mu, \quad p_A^{\dot{D}} p_B^A = -\delta_B^{\dot{D}} p_\mu p^\mu. \quad (27)$$

Thus solutions of equations (25), (26) fulfill the KDP equations (24). The spinor equations (25), (26) describe spin-1 bosons [13] where spinors η_{CA} , $\chi_{\dot{D}\dot{B}}$ correspond to selfdual or antiselfdual

antisymmetric tensors $\psi^{\mu\nu}$, respectively. Each of the above equations is covariant except from space reflection, but both equations taken together are fully covariant. These equations written in tensor form, $\beta^\mu p_\mu \Psi = m\Psi$, $\Psi = [\psi_{01}, \psi_{02}, \psi_{03}, \psi_0, \psi_1, \psi_2, \psi_3]^T$ where $\psi^{\mu\nu}$ are selfdual or antiselfdual antisymmetric tensors, with 7×7 matrices β^μ fulfilling equation (18), are the Hagen–Hurley equations [14, 4].

The spinor form of either of the Hagen–Hurley equations (25), (26) can be splitted again to obtain two 3×3 equations analogous to equations (9) and (10) and a Klein–Gordon equation [15].

Let us conclude with several general remarks. We have shown that spinor formalism discloses internal structure of KDP equations which manifests itself by presence of redundant components – there are special three-component solutions of these equations. Accordingly, the meson spin-0 and spin-1 KDP equations split into pairs of three-component constituent equations, each equation describing a particle with definite mass and partly undefined spin (all three-component constituent equations discussed above are similar in a sense that their matrices ρ^μ fulfill the same commutation relations (18) [11, 4]) and in the case of spin-1 KDP equations an additional wavefunction fulfilling the Klein–Gordon equation is present. Moreover, solutions of the constituent equations are subsolutions of appropriate Dirac equations and pairs of such Dirac equations, corresponding to pairs of constituent equations, are charge conjugated one to another. This last finding entitles us to conclude that Kemmer–Duffin–Petiau equations describe mesons as composed from quark-antiquark pairs. These results are consistent with quark theory of mesons [16].

Let us stress that the separation of a meson into constituents is imperfect, since although each of the constituent equations describes a massive particle, its Lorentz symmetry is broken. This offers explanation of quark confinement different than in quantum chromodynamics where linearly increasing potential energy between a quark and other quarks in a hadron is responsible for confinement [17]). We hope that our results can also cast light on the problem of spin crisis [18] since meson constituents in our theory have undefined spins. Let us assume that proton constituents (quarks) have the same nature as the meson constituents of our theory. It follows that the proton spin cannot be obtained as a sum of spins of its constituents since the constituents have (partly) undefined spins.

- [1] Corson E.M., Introduction to tensors, spinors and relativistic wave equations (relation structure), London, Blackie and Son, 1953.
- [2] Ghose P., Home D., and Sinha Roy M.N., Relativistic quantum mechanics of bosons, *Phys. Lett. A*, 1993, V.183, N 4, 267–271;
Ghose P. and Home D., On boson trajectories in the Bohm model, *Phys. Lett. A*, 1994, V.191, N 5–6, 362–364.
- [3] Lunardi J.T., Pimentel B.M., Teixeira R.G. and Valverde J.S., Remarks on Duffin–Kemmer–Petiau theory and gauge invariance, *Phys. Lett. A*, 2000, V.268, N 3, 165–173.
- [4] Beckers J., Debergh N. and Nikitin A.G., On Parasupersymmetries and relativistic descriptions for spin one particles: I. The free context, *Fortschr. Phys.*, 1995, V.43, N 1, 67–80;
Beckers J., Debergh N. and Nikitin A.G., On parasupersymmetries and relativistic descriptions for spin one particles: II. The interacting context with (electro)magnetic fields, *Fortschr. Phys.*, 1995, V.43, N 1, 81–96.
- [5] Nowakowski M., The electromagnetic coupling in Kemmer–Duffin–Petiau theory, *Phys. Lett. A*, V.244, N 5, 329–337.
- [6] Gonen S., Havare A. and Unal N., Exact solutions of Kemmer equation for Coulomb potential, hep-th/0207087.
- [7] Ghose P. and Samal M.K., EPR type nonlocality in classical electrodynamics, quant-ph/0111119.
- [8] Casana R., Fainberg V.Ya., Pimentel B.M., Lunardi J.T. and Teixeira R.G., Massless DKP fields in Riemann–Cartan space-times, *Class. Quantum Grav.*, 2003, V.20, N 11, 2457–2465; gr-qc/0209083.
- [9] Fushchich W.I. and Nikitin A.G., Poincaré-invariant equations of motion for particles with arbitrary spin, *Sov. J. Part. Nucl.*, 1978, V.9, N 3, 501–553 (in Russian);
Fushchich W.I. and Nikitin A.G., Symmetries of equations of quantum mechanics, New York, Allerton Press Inc., 1994.

- [10] Velo G. and Zwanziger D., Noncausality and other defects of interaction Lagrangians for particles with spin one and higher, *Phys. Rev.*, 1969, V.188, N 5, 2218–2226.
- [11] Okninski A., Effective quark equations, *Acta Physica Polonica B*, 1981, V.12, N 2, 87–94;
Okninski A., Dynamic theory of quark and meson fields, *Phys. Rev. D*, 1982, V.25, N 12, 3402–3407.
- [12] Bjorken J.D. and Drell S.D., Relativistic quantum mechanics, New York, McGraw-Hill, 1964.
- [13] Lopuszanski J., The representations of the Poincaré group in the framework of free quantum fields, *Fortschr. Phys.*, 1978, V.26, N 4, 261–288.
- [14] Hagen C.R. and Hurley W.J., Magnetic moment of a particle with arbitrary spin, *Phys. Rev. Lett.*, 1970, V.24, N 24, 1381–1384;
Hurley W.J., Relativistic wave equations for particles with arbitrary spin, *Phys. Rev. D*, 1971, V.4, N 12, 3605–3616.
- [15] Okninski A., On the inner structure of Kemmer–Duffin–Petiau equations, submitted.
- [16] Gell-Mann M. and Ne’eman Y., The eightfold way, New York, Benjamin, 1964.
- [17] Takahashi T.T., Matsufuru H., Nemoto Y. and Suganuma H., Three-quark potential in $SU(3)$ lattice QCD, *Phys. Rev. Lett.*, 2001, V.86, N 1, 18–21;
Takahashi T.T., Suganuma H., Nemoto Y. and Matsufuru H., Detailed analysis of the three quark potential in $SU(3)$ lattice QCD, *Phys. Rev. D*, 2002, V.65, N 11, 114509(19).
- [18] Ma B.-Q., Schmidt I. and Yang J.-J., Understanding the proton spin “puzzle” with a new “minimal” quark model, *Eur. Phys. J. A*, 2001, V.12, N 3, 353–359.