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D-Branes, Helices, and Proton Decay

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Proton decay is investigated by methods of category theory. The investigation leads to the conclusion that proton decay is forbidden.

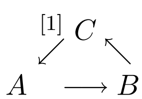
1 Introduction

Recently the new description of D-branes was proposed [1–3]. This description is based on methods of category theory [4]. In the present paper we apply these methods to investigation of proton decay.

2 The triangulated category

The triangulated category contains the following data [4]:

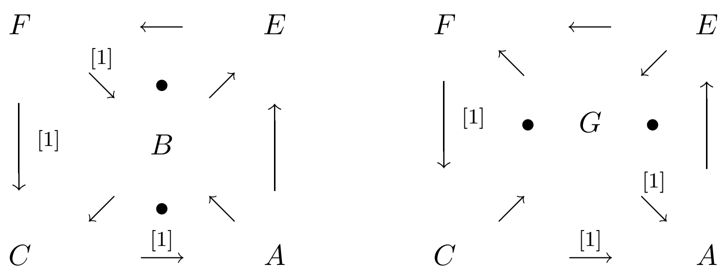
1) Distinguished triangles



$$C = \mathrm{Cone}(f)$$

(where vertices are complexes of coherent sheaves),

2) Octahedral diagrams



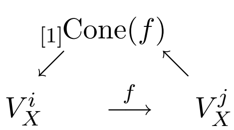
(1)

(where distinguished triangles are marked by •).

These data satisfy Verdier axioms.

3 Helices

Let us consider the special class of distinguished triangles



(2)

where V_X^i and V_X^j are coherent sheaves over the Calabi–Yau manifold X, which are constructed by mutations of helices [5–7].

A collection of coherent sheaves $\{\mathcal{R}_W^i\}$ over the weighted projective space W is called a helix if the following condition is satisfied: The Euler matrix

$$\chi\big(\mathcal{R}_W^i, \mathcal{R}_W^j\big) = \int_W \mathrm{ch}\big(\mathcal{R}_W^{i*} \otimes \mathcal{R}_W^j\big)\mathrm{td}(\mathcal{T}_W)$$

is an upper-triangular matrix with ones on the diagonal.

There exists a mutated helix $\{\mathcal{S}_W^j\}$ over the weighted projective space W if the following orthogonality relation holds

$$\int_W \mathrm{ch}\big(\mathcal{R}_W^i\big)\mathrm{ch}\big(\mathcal{S}_W^j\big)\mathrm{td}(\mathcal{T}_W) = \delta_{ij}.$$

Coherent sheaves V_X^j are obtained by the restriction of $\mathcal{S}_W^j$ to the Calabi–Yau manifold X.

We interpret vertices of distinguished triangles (2) as B-type D-branes if criteria for Π-stability are satisfied [2]. Edges of triangles (2) are interpreted as superstrings.

4 Π-stability

In order to investigate Π-stability of the D-brane Cone(f) against decay into the D-branes V_X^i and V_X^j we need to compute the central charges of V_X^i and V_X^j.

The central charge of V_X^i is determined by [2]

$$Z(V_X^j) = \sum_k Q_k^j \Pi^k = \int_X e^{-B-iJ}\mathrm{ch}\big(V_X^j\big)\sqrt{\mathrm{td}(\mathcal{T}_X)}, \tag{3}$$

where $Q_k^j \in H^3(Y,\mathbb{Z})$ are the RR charges [8] (Y is the mirror of X), Π^k is the Kähler period vector (which describes the Kähler moduli space of X [9]), $B+iJ$ is the complexified Kähler form, $\mathcal{T}_X$ is the tangent sheaf over X.

The grade associated with the central charge (3) is defined by

$$\varphi(V_X^j) = -\frac{1}{\pi}\arg Z\big(V_X^j\big)$$

The D-brane Cone(f) is Π-stable if

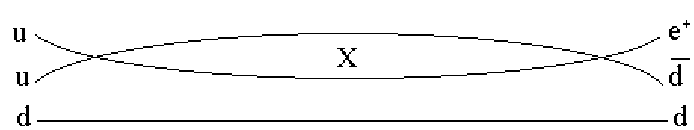
$$\varphi\big(V_X^j\big) - \varphi\big(V_X^i\big) < 0.$$

The application of criteria for Π-stability to distinguished triangles enclosed in the octahedral diagram (1) leads to the following rule of D-brane decays [2]:

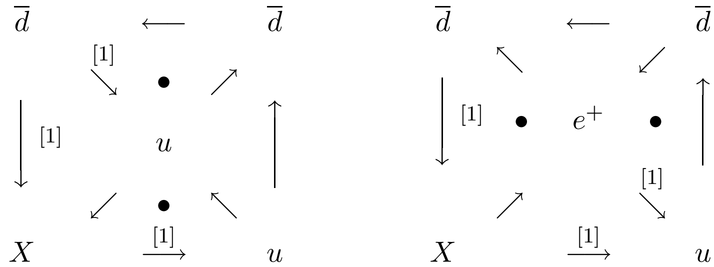
★ If C is stable against decay into A and B, but that B itself is unstable with respect to a decay into E and F, than C will always be unstable with respect to decay into F and some bound state G of A and E.

5 Proton decay

In a grand unified theory [10] proton decay is described by the quark-lepton diagram



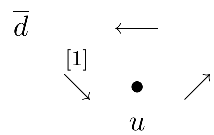
Assuming that quarks, leptons and X-bosons are solitonic excitations in a proton, we can construct the octahedral diagram



(4)

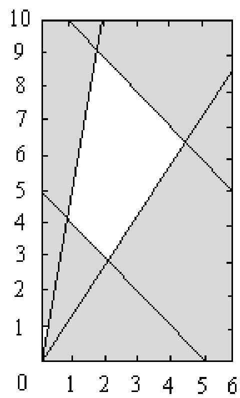
which induces proton decay.

Let us consider the distinguished triangle



(5)

enclosed in the octahedron (4). Taking into account the allowed region (shown in white) for u-quark and d-quark masses [11]



we conclude that in the triangle (5) u is stable with respect to a decay into $\overline{d}$ and $\overline{d}$. This conclusion is incompatible to the rule of decays ★ (where B is unstable with respect to a decay into E and F). Therefore proton decay is forbidden.

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