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# Classification of Equilibrium States of Condensed Media with Spontaneously Broken Symmetry

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In the present paper the classification of equilibrium states of condensed media with spontaneously broken symmetry is carried out. The used approach is based on concept of the quasiaverages. The admissible properties of equilibrium state symmetry of a quantum fluid and relevant structures of an order parameter are found from requirements of the residual and spatial symmetry at nonzero values of an order parameter. The superfluid in a state with triplet pairing, the superfluid nuclear matter in a state of $d$-pairing, liquid crystal states of matter are considered in this paper as the examples. Admissible conditions of the spatial symmetry and the general structure of the corresponding generator are found.

## 1 Introduction

The classification of equilibrium states of condensed media with spontaneously broken symmetry based on the phenomenological Ginzburg–Landau approach requires the exact free energy dependence on the order parameter function and essentially depends on the type of the model we consider. The other group-theory approach is based on the conception of residual symmetry of the degenerate state of equilibrium as the subgroup of normal phase symmetry. The corresponding transformation properties of the order parameter in Hamiltonian symmetry transformations are essential in this approach. This consideration is free from any model assumption about the form of free energy. The classification of homogenous states in terms of both mentioned approaches was carried out in particular for the superfluid $^3$He [1–3], and for $d$-pairing in superfluid quantum liquid [4, 5]. In the present paper the general microscopic approach to the classification of equilibrium states of condensed media with spontaneously broken symmetry based on the conception of quasiaverages is suggested. The permissible properties of symmetry of quantum liquid equilibrium state and the corresponding structures of order parameter are found from the conditions of residual symmetry at the nonzero values of order parameter. As the examples of degenerate condensed media with spontaneously broken symmetry we consider the superfluid liquids in state with triple type of pairing ($^3$He), superfluid nuclear matter in $d$-pairing state and liquid crystals.

## 2 Symmetry properties of the normal equilibrium state

The theory of many-particle systems that describes the equilibriums of a normal Fermi liquid is based on the statistical Gibbs operator

$$\hat{w} = \exp\left(\Omega - Y_a \hat{\gamma}_a\right), \tag{1}$$

here $\hat{\gamma}_a \equiv \left(\hat{\mathcal{H}}, \hat{\mathcal{P}}_k, \hat{N}, \hat{S}_\alpha\right)$ are additive integrals of motion ($a \equiv 0, k, 4, \alpha$): $\hat{\mathcal{H}}$ is the Hamiltonian, $\hat{\mathcal{P}}$ is the momentum operator, $\hat{N}$ is the particle number operator, $\hat{S}_\alpha$ is the spin operator, Thermodynamic potential $\Omega = V\omega(Y)$ is defined by the normalization condition $\mathrm{Sp}\,\hat{w} = 1$. The set of thermodynamic forces $Y_a$ includes the temperature $Y_0^{-1} \equiv T$, the velocity $Y_k^{-1} \equiv v_k$, the chemical potential $-Y_4/Y_0 \equiv \mu_k$, and the effective magnetic field $-Y_\alpha/Y_0 \equiv h_\alpha$.

The additive integrals of motion entering the Gibbs distribution result in a certain symmetry of the equilibrium.The symmetry properties of equilibrium statistical operator (1) have the form

$$\big[\hat{w}, \hat{\mathcal{P}}_k\big] = 0, \qquad \big[\hat{w}, \hat{\mathcal{H}}\big] = 0, \qquad \big[\hat{w}, \hat{N}\big] = 0, \qquad \big[\hat{w}, \hat{\Sigma}_\alpha\big] = 0, \qquad \big[\hat{w}, \hat{L}_k\big] = 0 \tag{2}$$

and reflect the space-time translation invariance and the phase invariance. The requirement for the symmetry under rotations in the spin and configuration spaces implies neglecting the weak dipole and spin-orbit interactions. Here, $\hat{\Sigma}_\alpha$ and $\hat{L}_i$ are the generalized spin and orbital momentum operators

$$\hat{\Sigma}_\alpha \equiv \hat{S}_\alpha + \hat{S}^Y_\alpha, \qquad \hat{S}^Y_\alpha \equiv -i\varepsilon_{\alpha\beta\gamma} Y_\beta \frac{\partial}{\partial Y_\gamma}, \qquad \hat{L}_i \equiv \hat{\mathcal{L}}_i + \hat{\mathcal{L}}^Y_i, \qquad \hat{\mathcal{L}}^Y_i \equiv -i\varepsilon_{ikl} Y_k \frac{\partial}{\partial Y_l}, \tag{3}$$

acting on the Hilbert space and on the thermodynamic parameter space. The action of the differential operators on the vectors $Y_i(Y_\alpha)$ is defined by $i\big[\hat{\mathcal{L}}^Y_i, Y_j\big] = \varepsilon_{ikj} Y_k$, $i\big[\hat{S}^Y_\alpha, Y_\rho\big] = \varepsilon_{\alpha\beta\rho} Y_\beta$. The corresponding means of commutators involving the orbital momentum operators have the form $\mathrm{Sp}\,\big[\hat{w}, \hat{\mathcal{L}}_i + \mathcal{L}^Y_i\big]\hat{b}(x) = \mathrm{Sp}\,\hat{w}\big[\hat{\mathcal{L}}_i, \hat{b}(x)\big] + \mathcal{L}^Y_i \mathrm{Sp}\,\hat{w}\hat{b}(x)$. According to definition (3), the operators $\hat{\Sigma}_\alpha$ and $\hat{L}_i$ satisfy the relations

$$i\big[\hat{L}_i, \hat{L}_k\big] = -\varepsilon_{ikl}\hat{L}_l, \qquad i\big[\hat{\Sigma}_\alpha, \hat{\Sigma}_\beta\big] = -\varepsilon_{\alpha\beta\gamma}\hat{\Sigma}_\gamma.$$

## 3 Equilibrium degeneracy and quasiaverages

The theoretical foundation of the statistical physics that describes equilibriums of condensed matter with a spontaneously broken symmetry is Bogoliubov's concept of quasiaverages [6]. A constructive point of this concept is inserting an infinitely small source $\nu\hat{F}$ into the equilibrium statistical operator, that reduces the symmetry of the statistical equilibrium in comparison with the symmetry of the Hamiltonian and allows generalizing the Gibbs distribution to condensed matter under spontaneous symmetry-breaking conditions. The quasiaverage value of a quantity $a(x)$ in a statistical equilibrium with a broken symmetry is defined by

$$\langle \hat{a}(x)\rangle \equiv \lim_{\nu\to 0}\lim_{V\to\infty} \mathrm{Sp}\,\hat{w}_\nu \hat{a}\,(x)\,, \qquad \hat{w}_\nu \equiv \exp\big(\Omega_\nu - Y_a\hat{\gamma}_a - \nu\hat{F}\big). \tag{4}$$

The operator $\hat{F}$ has the symmetry of the condensed-matter phase under study and removes the degeneracy of the equilibrium. In accordance with the concept of quasiaverages, we choose the source $\hat{F}$ in the form of a linear functional of an order parameter $\hat{\Delta}_a\,(x)$

$$\hat{F} = \int d^3x\big(f_a(x,t)\hat{\Delta}_a(x) + \mathrm{h.c.}\big),$$

where $f_a(x,t)$ is some function of coordinates and time that is conjugate to the order parameter and defines its equilibrium value $\Delta_a(x,t) = \langle\hat{\Delta}_a(x)\rangle$ in the sense of quasiaverages (4). The structure of the function $f_a(x,t)$ is defined by the symmetry properties of the phase under study. The last circumstance gives a possibility of introducing additional thermodynamic parameters into the Gibbs distribution in the framework of the microscopic theory [7].

A description of condensed matter with a spontaneously broken symmetry essentially rests on the notion of the order parameter. In the language of the secondary quantization, the order-parameter operators $\hat{\Delta}_a(x)$ are built from the field creation and annihilation operators. We form late the commutation properties of the order-parameter operators. The translation invariance condition is

$$i\big[\hat{\mathcal{P}}_k, \hat{\Delta}_a(x)\big] = -\nabla_k \hat{\Delta}_a(x). \tag{5}$$

The generator of the phase transformation group is the particle number operator $\hat{N}$. The relation

$$\big[\hat{N}, \hat{\Delta}_a(x)\big] = -g_a \hat{\Delta}_a(x) \tag{6}$$

holds for the order-parameter operator $\hat{\Delta}_a(x)$. The constants $g_a$ depend on the tensor dimension of the order-parameter operator.

Under the transformations related to the internal-symmetry group with the generators $\hat{S}_\alpha$ $(\alpha = x, y, z)$ the operators $\hat{\Delta}_a(x)$ transform according to the representations of this group, which yields the equality

$$i\big[\hat{S}_\alpha, \hat{\Delta}_a(x)\big] = -g_{\alpha ab} \hat{\Delta}_b(x), \tag{7}$$

or, in compact notation,

$$i\big[\hat{S}_\alpha, \hat{\Delta}(x)\big] = -\hat{g}_\alpha \hat{\Delta}(x),$$

where $(\hat{g}_\alpha)_{ab} \equiv g_{\alpha ab}$– are some constants. Because $i\big[\hat{S}_\alpha, \hat{S}_\beta\big] = -\varepsilon_{\alpha\beta\gamma}\hat{S}_\gamma$, from formula (7) and the Jacobi identity for the operators $\hat{S}_\alpha$ and $\hat{\Delta}\,(x)$, we have

$$\big[\hat{g}_\alpha, \hat{g}_\beta\big] = -\varepsilon_{\alpha\beta\gamma}\hat{g}_\gamma. \tag{8}$$

Under the transformations related to the spatial-rotation group with the generators $\mathcal{L}_i$ $(i = 1, 2, 3)$, the order parameter operators $\hat{\Delta}_a(x)$ at the point $x = 0$ transform according to the representations of this group. The equality $i\big[\hat{\mathcal{L}}_i, \hat{\Delta}_a(0)\big] = -g_{iab}\hat{\Delta}_b(0)$ therefore holds, and noting that $\big[\hat{\mathcal{L}}_i, \hat{\mathcal{L}}_j\big] = i\varepsilon_{ijk}\hat{\mathcal{L}}_k$, we obtain a relation similar to (8)

$$\big[\hat{g}_i, \hat{g}_j\big] = -\varepsilon_{ijk}\hat{g}_k.$$

Because $\hat{\Delta}_a(x) = e^{-i\hat{\mathcal{P}}x}\hat{\Delta}_a(0)e^{i\hat{\mathcal{P}}x}$, $e^{-i\hat{\mathcal{P}}x}\hat{\mathcal{L}}_i e^{i\hat{\mathcal{P}}x} = \hat{\mathcal{L}}_i - \varepsilon_{ijk}x_j\hat{\mathcal{P}}_k$, by virtue of (5) we have

$$i\big[\hat{\mathcal{L}}_i, \hat{\Delta}_a(x)\big] = -g_{iab}\hat{\Delta}_b(x) - \varepsilon_{ijk}x_k\nabla_j\hat{\Delta}_a(x). \tag{9}$$

It is known from the phenomenological theory that, in general, an adequate description of the thermodynamics of non-equilibrium processes in condensed matter with a broken symmetry requires introducing new thermodynamic parameters into the theory; these are not related to conservation laws but are due to the physical nature of the thermodynamic phase. In the case of normal condensed matter, the thermodynamic parameters are defined by only the densities of the additive integrals of motion. We show how the symmetry properties are formulated for the equilibriums of degenerate condensed matter and how the additional thermodynamic parameters are introduced based on this. We consider the translation-invariant subgroups of the residual symmetry $H$ of the total group $G$. The translation invariance implies that the equilibrium statistical operator satisfies the symmetry relation

$$\big[\hat{w}, \hat{\mathcal{P}}_k\big] = 0. \tag{10}$$

We analyze the translation-invariant subgroups of the residual symmetry for the equilibriums starting from the relation

$$\big[\hat{w}, \hat{T}\big] = 0, \tag{11}$$

where the generator $T$ of the residual symmetry (a generator of the subgroup $H$) is a linear combination of the integrals of motion (see for comparison (2))

$$\hat{T} \equiv a_i \hat{L}_i + b_\alpha \hat{\Sigma}_\alpha + c\hat{N} \equiv \hat{T}(\xi) \tag{12}$$

with some real parameters $(a_i, b_\alpha, c \equiv \xi)$. The unitary transformations $U(\xi) = \exp[i\hat{T}(\xi)]$ form the continuous subgroup of the residual symmetry $U(\xi)U(\xi') = U(\xi''(\xi, \xi'))$ for the equilibrium state. Using the equalities

$$i\,\mathrm{Sp}\big[\hat{w}, \hat{T}\big]\hat{\Delta}_a(x) = 0, \qquad i\,\mathrm{Sp}\big[\hat{w}, \hat{\mathcal{P}}_k\big]\hat{\Delta}_a(x) = 0,$$

and taking algebraic relations (5)–(7), (9) and definition (12) into account, we obtain the equations

$$a_i\left(g_{iab}\Delta_b + \varepsilon_{ikl}Y_k\frac{\partial\Delta_a}{\partial Y_l}\right) + b_\alpha\left(g_{\alpha ab}\Delta_b + \varepsilon_{\alpha\beta\gamma}Y_\beta\frac{\partial\Delta_a}{\partial Y_\gamma}\right) + ig_a\Delta_a = 0, \qquad \nabla_k\Delta_a = 0.$$

which establish certain relations between the parameters $\xi$. For simplicity, we consider the case where $Y_\alpha = Y_k = 0$. In this case, we have

$$T_{ab}\Delta_b = 0, \qquad T_{ab} \equiv a_i g_{iab} + b_\alpha g_{\alpha ab} + ig_a\delta_{ab}. \tag{13}$$

The requirement that set of linear equations (13) has a nontrivial solution $\Delta_a \neq 0$ leads to the equality $\det|T_{ab}| = 0$, which imposes restrictions on the admissible values of the parameters $\xi$ of the residual symmetry generator.

## 4 Translation-invariant equilibrium states of the condensed media with spontaneously broken symmetry

A. *Translation-invariant equilibrium states of the superfluid* $^3$He. For the order parameter operator $\hat{\Delta}_{\alpha k}(x)$ it is convenient to choose [7]

$$\hat{\Delta}_{\alpha k}(x) \equiv \hat{\psi}(x)\,\sigma_2\sigma_\alpha\nabla_k\hat{\psi}(x) - \nabla_k\hat{\psi}(x)\,\sigma_2\sigma_\alpha\hat{\psi}(x)\,.$$

Here, $\sigma_\alpha$ are Pauli matrices. The matrices $(\sigma_2\sigma_\alpha)_{\mu\nu} = (\sigma_2\sigma_\alpha)_{\nu\mu}$ are symmetric with respect to indices $\mu$ and $\nu$. We see that in accordance with this definition and the canonical commutation relations for Fermi operators, the equalities are valid:

$$\begin{aligned} &i\big[\hat{S}_\alpha, \hat{\Delta}_{\beta i}(x)\big] = -\varepsilon_{\alpha\beta\gamma}\hat{\Delta}_{\gamma i}(x), \qquad \big[\hat{N}, \hat{\Delta}_{\beta i}(x)\big] = -2\hat{\Delta}_{\beta i}(x),\\ &i\big[\hat{\mathcal{P}}_k, \hat{\Delta}_{\alpha i}(x)\big] = -\nabla_k\hat{\Delta}_{\alpha i}(x), \qquad i\big[\hat{\mathcal{L}}_k, \hat{\Delta}_{\alpha i}(x)\big] = -\varepsilon_{kjl}x_j\nabla_l\hat{\Delta}_{\alpha i}(x) - \varepsilon_{kil}\hat{\Delta}_{\alpha l}(x). \end{aligned} \tag{14}$$

By virtue of algebra (14) and relations (13), we obtain the equality defining the equilibrium structure of the order parameter:

$$a_k\varepsilon_{kil}\Delta_{\beta l} + b_\alpha\varepsilon_{\alpha\beta\gamma}\Delta_{\gamma i} + 2ic\Delta_{\beta i} = 0.$$

This system of equations has many anisotropic solutions that are present in Table 1. Let us consider the state corresponding to the isotropic superfluid phase. We introduce the orthogonal rotation matrix that describes the change of the orientation of the spatial coordinate system with respect to the spin $S = 1$ by the equality $b_\alpha = a_i R_{i\alpha}$. Taking (12) into account, we obtain $a_i\big[\hat{w}, \hat{\mathcal{L}}_i + R_{i\alpha}\hat{S}_\alpha\big] = 0$. The isotropy condition implies the validity of the last relation for arbitrary directions of the vector $\vec{a}$. The symmetry property of the states therefore has the form

$$\big[\hat{w}, \hat{\mathcal{L}}_i + R_{i\alpha}\hat{S}_\alpha\big] = 0.$$

This state describes the $B$ phase of the superfluid $^3$He. For the states with this symmetry, the mean of the order parameter is

$$\Delta_{\alpha k} = \Delta R_{k\alpha}.$$

The resulting classification of the residual symmetry properties and the corresponding order parameter values in the equilibrium of $^3$He for the translation-invariant case is presented in Table 1. We assume that the projection of the Cooper-pair orbital momentum onto the $\vec{l}$ direction is equal to $m_l$ and the projection of the Cooper-pair spin onto the $\vec{d}$ direction is equal to $m_s$.

**Table 1.**

| Residual symmetry generator | $m_l$ | $m_s$ | Order parameter | Phase |
|---|---|---|---|---|
| $\hat{\mathcal{L}}_i + R_{i\alpha}\hat{S}_\alpha$ | – | – | $\Delta R_{\alpha i}$ | $B$ |
| $\vec{l}\hat{\vec{\mathcal{L}}} - \frac{m_l}{2}\hat{N}$ | $\pm1$ | $0$ | $\Delta d_\alpha\left(m_k \mp i n_k\right)$ | $A$ |
| | $0$ | $\pm1$ | $\Delta\left(e_\alpha \mp i f_\alpha\right) l_k$ | $\beta$ |
| $\vec{d}\hat{\vec{S}} - \frac{m_s}{2}\hat{N}$ | $\pm1$ | $\pm1$ | $\Delta\left(e_\alpha \mp i f_\alpha\right)\left(m_k \mp i n_k\right)$ | $A_1$ |
| | $0$ | $0$ | $\Delta d_\alpha l_k$ | Polar |
| $\vec{d}\hat{\vec{S}} - 2m_l\left\|m_s\right\|\vec{l}\hat{\vec{\mathcal{L}}} - \frac{1}{2}m_s\hat{N}$ | $0, \pm1$ | $0$ | $d_\alpha\left(A m_k + B n_k + C l_k\right)$ | |
| | $\pm1$ | $\pm1$ | $A\left(m_k \mp i n_k\right)\left(e_\alpha \mp i f_\alpha\right)$ $+B d_\alpha\left(m_k \mp i n_k\right)$ | $A_1 + A$ |
| | $0$ | $\pm1$ | $\left(e_\alpha \mp i f_\alpha\right)\left(A m_k + B n_k + C l_k\right)$ | $A_2$ |
| $\vec{l}\hat{\vec{\mathcal{L}}} - 2m_s\left\|m_l\right\|\vec{d}\hat{\vec{S}} - \frac{1}{2}m_l\hat{N}$ | $0$ | $0, \pm1$ | $\left(A e_\alpha + B f_\alpha + C d_\alpha\right) l_k$ | |
| | $\pm1$ | $\pm1$ | $A\left(m_k \mp i n_k\right)\left(e_\alpha \mp i f_\alpha\right) + B l_k\left(e_\alpha \mp i f_\alpha\right)$ | $A_1 + \beta$ |
| | $\pm$ | $0$ | $\left(A e_\alpha + B f_\alpha + C d_\alpha\right)\left(m_k \mp i n_k\right)$ | $B_2$ |
| $\vec{l}\hat{\vec{\mathcal{L}}} + \vec{d}\hat{\vec{S}} - \frac{1}{2}(m_l + m_s)\hat{N}$ | $0, \pm$ | $0, \mp$ | $e_\alpha\left(A m_k + B n_k\right) + f_\alpha\left(-B m_k + A n_k\right)$ $+C d_\alpha l_k$ | $\zeta$ |
| | $0$ | $\mp$ | | |
| | | | $A l_k\left(e_\alpha \mp i f_\alpha\right) + B d_\alpha\left(m_k \mp i n_k\right)$ | $\epsilon$ |
| | $\pm$ | $0$ | | |
| | $\pm$ | $\pm$ | $\Delta\left(e_\alpha \mp i f_\alpha\right)\left(m_k \mp i n_k\right)$ | $A_1$ |

B. *The superfluid nuclear matter in a state of d-pairing.* We define the order parameter operator of $d$-pairing in the terms of the operators of creation and annihilation of Fermi-particle:

$$\hat{\Delta}_{ik}(x) \equiv \nabla_i\hat{\psi}(x)\sigma_2\nabla_k\hat{\psi}(x) + \nabla_k\hat{\psi}(x)\sigma_2\nabla_i\hat{\psi}(x) - \frac{2}{3}\delta_{ik}\nabla_j\hat{\psi}(x)\sigma_2\nabla_j\hat{\psi}(x), \tag{15}$$

where $\sigma_2$ is the Pauli matrix.

Using this definitions we obtain the operator relations for the operators of number of particles $\hat{N}$, momentum $\hat{\mathcal{P}}_k$, spin $\hat{S}_\alpha$ and orbital momentum $\hat{\mathcal{L}}_k$

$$\begin{gathered}\left[\hat{N}, \hat{\Delta}_{ik}(x)\right] = -2\hat{\Delta}_{ik}(x), \qquad \left[\hat{S}_\alpha, \hat{\Delta}_{ik}(x)\right] = 0, \qquad i\left[\hat{\mathcal{P}}_l, \hat{\Delta}_{ik}(x)\right] = -\nabla_l\hat{\Delta}_{ik}(x), \\ i\left[\hat{\mathcal{L}}_l, \hat{\Delta}_{ik}(x)\right] = -\varepsilon_{lij}\hat{\Delta}_{jk}(x) - \varepsilon_{lkj}\hat{\Delta}_{ji}(x) - \varepsilon_{lkj}x_k\nabla_j\hat{\Delta}_{ik}(x)\end{gathered} \tag{16}$$

The mean values of the order parameter $\Delta_{ik}(x,\hat{\rho}) = \mathrm{Sp}\,\hat{\rho}\hat{\Delta}_{ik}(x)$ , ($\hat{\rho}$ is the arbitrary statistical operator) have properties $\Delta_{ik}(x,\hat{\rho}) = \Delta_{ki}(x,\hat{\rho})$, $\Delta_{ii}(x,\hat{\rho}) = 0$.

If we use the general approach developed in Section 3 we get the relation for residual symmetry operator (12)

$$\left[\hat{w}, \frac{a_i}{a}\hat{\mathcal{L}}_i - \frac{M}{2}\hat{N}\right] = 0. \tag{17}$$

Here $M$ is quantum number taking the values 0, $\pm 1$, $\pm 2$. Let us decompose the vector $\vec{a}$ on the indicated coordinate $\vec{a}/a = \alpha\vec{n} + \beta\vec{m} + \gamma\vec{l}$. The vectors $\vec{m}$, $\vec{n}$, $\vec{l}$ are the axes of an anisotropy, representative unitary and mutually perpendicular vectors and the quantities $\alpha$, $\beta$, $\gamma$ obey the equality $\alpha^2 + \beta^2 + \gamma^2 = 1$.

From this relation it is easy to obtain the expression for the order parameter, according to formalism developed in Section 3 and taking into account (15), (16). To obtain the final expression for order parameter we define mean value according to the paper [8]

$$\Delta \equiv k_i \Delta_{ij} k_j, \qquad \vec{k} \equiv \vec{m}\sin\theta\sin\varphi + \vec{n}\cos\theta + \vec{l}\sin\theta\cos\varphi.$$

For $M = 0$ in according to (13), (17) we obtain

$$\Delta_1^{(0)} = iE\left\{(\beta k_y + \alpha k_z + \gamma k_x)^2 - \frac{1}{3}\right\}, \qquad \Delta_2^{(0)} = i\widetilde{C}\left\{(\beta k_y + \alpha k_z)^2 - \frac{1}{3}\right\},$$
$$\Delta_3^{(0)} = (A + iC)\left(k_z^2 - 1/3\right), \qquad E^2 = (3/2), \qquad \widetilde{C^2} = (3/2), \qquad A^2 + C^2 = 3/2.$$

These solutions correspond to the "real" state [8].

The solution for $M = \pm 1$ in according to (15), (17) gives the equality

$$\Delta^{(1)} = A(k_x + k_y)\big(\pm i\sqrt{2}k_z + k_y - k_x\big), \qquad A^2 = 1/4.$$

At last, in a case ($M = \pm 2$) we obtain

$$\Delta^{(2)} = A(k_x \pm ik_y)^2, \qquad A^2 = 1/4.$$

This solution coincides with "axial" state of work [8].

C. *Liquid crystal states of matter*. Let us consider the translation invariant states of liquid crystal equilibrium and define their possible equilibrium structures of order parameter. We choose the order parameter in the form

$$\hat{Q}_{uv}(x) \equiv \nabla_u\hat{\psi}^+(x)\nabla_v\hat{\psi}(x) + \nabla_v\psi^+(x)\nabla_u\hat{\psi}(x) - \frac{2}{3}\delta_{uv}\nabla_j\hat{\psi}^+(x)\nabla_j\hat{\psi}(x). \tag{18}$$

According to definition (1) of operators for number of particles, momentum operator and orbital momentum, we obtain the commutative relations:

$$\big[\hat{N}, \hat{Q}_{uv}(x)\big] = 0, \qquad i\big[\hat{\mathcal{P}}_k, \hat{Q}_{uv}(x)\big] = -\nabla_k\hat{Q}_{uv}(x),$$
$$i\big[\hat{\mathcal{L}}_i, \hat{Q}_{uv}(x)\big] = -\varepsilon_{iuj}\hat{Q}_{jv}(x) - \varepsilon_{ivj}\hat{Q}_{ju}(x) - \varepsilon_{ikl}x_k\nabla_l\hat{Q}_{uv}(x). \tag{19}$$

In case of uniaxial nematic we perform the analysis of the subgroups of unbroken symmetry states in the rest frame reference on the assumption of (11). As the phase invariance in liquid crystal is not broken, the expression for residual generator for homogeneous state has the form

$$\hat{T} \equiv a_i\hat{\mathcal{L}}_i,$$

here $a_i$ are the real parameters. According to (18), (19), (11) we have

$$Q_{ik} = Q\left(n_i n_k - \frac{1}{3}\delta_{ik}\right).$$

In the case of biaxial nematic the residual symmetry generator has the form

$$\hat{T}\big(\vec{a}, \vec{m}, \vec{n}\big) \equiv a_i\hat{L}_i\big(\vec{m}, \vec{n}\big). \tag{20}$$

The generalized orbital momentum operator is defined by equality

$$\hat{L}_i(\vec{m},\vec{n}) \equiv \hat{\mathcal{L}}_i + \hat{\mathcal{L}}_i^{\vec{m}} + \hat{\mathcal{L}}_i^{\hat{n}}, \qquad \hat{\mathcal{L}}_i^{\vec{m}} \equiv -i\varepsilon_{ikl} m_k \frac{\partial}{\partial m_l}, \qquad \hat{\mathcal{L}}_i^{\vec{n}} \equiv -i\varepsilon_{ikl} n_k \frac{\partial}{\partial n_l}. \tag{21}$$

The vectors $\vec{n}$ and $\vec{m}$ are unitary and mutually perpendicular vectors. Using the condition of symmetry (11) with the generator in the form (20), (21) we obtain the following order parameter

$$Q_{ik} = Q_1 \left( n_i n_k - \frac{1}{3}\delta_{ik} \right) + Q_2 \left( m_i m_k - \frac{1}{3}\delta_{ik} \right).$$

The description of the equilibrium state of degenerate condensed media with rather complicated order parameter is considered on the basis of microscopic approach. Representation of residual symmetry of the equilibrium state in combination with the conception of quasiaverages allows to suggest an alternative to the Ginzburg–Landau method of classification of possible condensed media phase states. The present approach can be generalized for description of the states without translation invariance. It gives the possibility of consideration of the periodical spatial structures with discrete or continuous residual symmetry.

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