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D-branes on Calabi–Yau Hypersurfaces

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The construction of BPS D-branes of the type IIB superstring theory on two- and three-parameter elliptic and $K3$ fibered Calabi–Yau hypersurfaces is discussed. The explicit maps between the characteristic classes of the Chan–Paton fiber bundle and the D-brane charges is established.

1 Introduction

In this note we study the description of BPS D-branes of the type IIB superstring theory compactified on Calabi–Yau manifolds. It is well know that superstring theory is not only theory of superstrings but also another of extended objects, D-branes, which although are not perturbative string states themselves but allow description in the perturbative string theory.

In spite of significant progress in the study of D-branes on the curved space-time spaces, in particular, on Calabi–Yau manifolds, situation is far from being understood. In the arsenal of the theoretical physics there is a number of simple Calabi–Yau manifolds, which are convenient for studying superstring theories and their D-branes. In this note we consider D-branes on the Calabi–Yau hypersurfaces of weighted projective spaces with two- and three-parameter Kähler moduli spaces. These surfaces have orbifold singularities. After resolving these singularities the hypersurfaces exhibit structures of fibered Calabi–Yau manifolds. It helps us to make the description of D-branes on these manifolds in more detail. The analysis of string dualities in four space-time dimensions requires to research D-branes wrapped on supersymmetric cycles of compactifying manifold. The study of boundary states of D-branes wrapped around of supersymmetric cycles in the quintic hypersurface of four dimension projective space was done in [1]. The detailed information on a part of spectrum of the D-branes on Calabi–Yau manifold in the geometric phase was presented in [2]. In [3] the relevant aspects were extended to some other models.

2 Weighted projective spaces

We consider weighted projective spaces $P^4_{(k_1,k_2,k_3,k_4,k_5)}$ with weights (k_1,k_2,k_3,k_4,k_5) described in five complex "homogeneous coordinates" (z_1,z_2,z_3,z_4,z_5) not all of them vanishing, which are subject to identification

$$(z_1,z_2,z_3,z_4,z_5) \simeq \left(\lambda^{k_1}z_1,\lambda^{k_2}z_2,\lambda^{k_3}z_3,\lambda^{k_4}z_4,\lambda^{k_5}z_5\right) \tag{1}$$

for nonzero λ. Thus, a weighted projective space is a generalization of ordinary projective space and in this notation $P^4 = P^4_{(1,1,1,1,1)}$. These spaces, except of $P^4_{(k_1,k_2,k_3,k_4,k_5)} = P^4_{(1,1,1,1,1)}$, have orbifold singularities due to the identification (1).

Indeed, if we put $z_i = (\zeta_i)^{k_i}$ then from relation (1) we obtain

$$(\zeta_1^{k_1},\zeta_2^{k_2},\zeta_3^{k_3},\zeta_4^{k_4},\zeta_5^{k_5}) \simeq \left((\lambda\zeta_1)^{k_1},(\lambda\zeta_2)^{k_2},(\lambda\zeta_3)^{k_3},(\lambda\zeta_4)^{k_4},(\lambda\zeta_5)^{k_5}\right) \tag{2}$$

that is

$$\zeta_i^{k_i} \simeq (\lambda\zeta_i)^{k_i},$$

or

$$(\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5) \simeq \lambda\,(\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5)$$

and

$$(\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5) \simeq \left(\zeta_1 e^{\frac{2\pi i}{k_1}},\zeta_2 e^{\frac{2\pi i}{k_2}},\zeta_3 e^{\frac{2\pi i}{k_3}},\zeta_4 e^{\frac{2\pi i}{k_4}},\zeta_5 e^{\frac{2\pi i}{k_5}}\right).$$

Therefore,

$$P^4_{(k_1,k_2,k_3,k_4,k_5)} = \frac{P^4_{(1,1,1,1,1)}}{Z_{k_1}\times Z_{k_2}\times Z_{k_3}\times Z_{k_4}\times Z_{k_5}}$$

and we have obtained a manifold $P^4_{(k_1,k_2,k_3,k_4,k_5)}[d], d = k_1+k_2+k_3+k_4+k_5$, with orbifold singularities.

We wish to study D-branes on the simplest Calabi–Yau hypersurfaces defined by polynomials on the homogeneous coordinates in weighted projective spaces.

The one-parameter Calabi–Yau hypersurfaces M [4] are given by

$$z_1^5+z_2^5+z_3^5+z_4^5+z_5^5=0, \qquad (z_1:z_2:z_3:z_4:z_5)\in P^4_{(1,1,1,1,1)}[5], \tag{3}$$

$$z_1^6+z_2^6+z_3^6+z_4^6+z_5^3=0, \qquad (z_1:z_2:z_3:z_4:z_5)\in P^4_{(1,1,1,1,2)}[6], \tag{4}$$

$$z_1^8+z_2^8+z_3^8+z_4^8+z_5^2=0, \qquad (z_1:z_2:z_3:z_4:z_5)\in P^4_{(1,1,1,1,4)}[8], \tag{5}$$

$$z_1^{10}+z_2^{10}+z_3^{10}+z_4^5+z_5^2=0, \qquad (z_1:z_2:z_3:z_4:z_5)\in P^4_{(1,1,1,2,5)}[10]. \tag{6}$$

In order to study the stringy geometry of these manifolds we consider one-parameter family of mirror manifolds W given by the Greene–Plesser orbifold construction $\{p_i=0\}/G_i$

$$z_1^5+z_2^5+z_3^5+z_4^5+z_5^5-5\psi z_1z_2z_3z_4z_5=0, \qquad G=Z_5\times Z_5, \tag{7}$$

$$z_1^6+z_2^6+z_3^6+z_4^6+z_5^3-6\psi z_1z_2z_3z_4z_5=0, \qquad G=Z_3\times Z_6^2, \tag{8}$$

$$z_1^8+z_2^8+z_3^8+z_4^8+z_5^2-8\psi z_1z_2z_3z_4z_5=0, \qquad G=Z_2\times Z_8^2, \tag{9}$$

$$z_1^{10}+z_2^{10}+z_3^{10}+z_4^5+z_5^2-10\psi z_1z_2z_3z_4z_5=0, \qquad G=Z_{10}^2. \tag{10}$$

The stringy geometry of the Calabi–Yau hypersurfaces M [5,6].

$$8z_1^8+z_2^8+z_3^4+z_4^4+z_5^4=0, \qquad (z_1:z_2:z_3:z_4:z_5)\in P^4_{(1,1,2,2,2)}, \tag{11}$$

$$z_1^{12}+z_2^{12}+z_3^6+z_4^6+z_5^2=0, \qquad (z_1:z_2:z_3:z_4:z_5)\in P^4_{1,1,2,2,6}, \tag{12}$$

$$z_1^{18}+z_2^{18}+z_3^{18}+z_4^3+z_5^2=0, \qquad (z_1:z_2:z_3:z_4:z_5)\in P^4_{1,1,1,6,9}. \tag{13}$$

are defined by the two-parameter families of the mirror manifolds W obtained by the Greene–Plesser orbifold construction $\{p_i=0\}/G$.

$$z_1^8+z_2^8+z_3^4+z_4^4+z_5^4-8\psi z_1z_2z_3z_4z_5-2\phi z_1^4z_2^4=0, \qquad G=Z_4^3, \tag{14}$$

$$z_1^{12}+z_2^{12}+z_3^6+z_4^6+z_5^2-12\psi z_1z_2z_3z_4z_5-2\phi z_1^6z_2^6=0, \qquad G=Z_6^2\times Z_2, \tag{15}$$

$$z_1^{18}+z_2^{18}+z_3^{18}+z_4^3+z_5^2-18\psi z_1z_2z_3z_4z_5-3\phi x_1x_2x_3=0, \qquad G=Z_{18}. \tag{16}$$

The Calabi–Yau hypersurface M defined by the equation

$$z_1^{24}+z_2^{24}+z_3^{12}+z_4^3+z_5^2=0, \qquad (z_1:z_2:z_3:z_4:z_5)\in P^4_{(1,1,2,8,12)} \tag{17}$$

has its three-parameter mirror family of W obtained by the Greene–Pleser orbifold construction

$$\begin{aligned} &z_1^{24}+z_2^{24}+z_3^{12}+z_4^3+z_5^2-12\psi_0 z_1z_2z_3z_4z_5\\ &\qquad -2\psi_1(z_3z_4z_5)^6-\psi_2(z_4z_5)^{12}=0, \qquad G=Z_6\times Z_{12}. \end{aligned} \tag{18}$$

3 Classical geometry of elliptic and $K3$ fibrations

The hypersurface $P^4_{(1,1,1,6,9)}[18]$ has two divisors E and L, which generate $H_4(M,Z)$. The exceptional divisor E in M is generated by blowing up singular line $z_2 = z_3 = 0$, which intersects M in a single point. The divisor L is defined by first order polynomial in z_1, z_2 and z_3.

The two homology classes h, l generate homology group $H_2(M,Z)$. The elliptic fibration structure is induced by linear system $|L|$ which maps M to P^2. The generic fiber can be proved to be an elliptic curve. The homology class l is a hyperplane class of E.

The choice (E,L) as generators of the complexified Kähler cone leads to generic Kähler class $K = t_1E + t_2L$, where (t_1,t_2) are classical coordinates on Kähler moduli space of M.

The models $P^4_{(1,1,2,2,2)}[8]$ and $P^4_{(1,1.2,2,6)}[12]$ given by degree 8 and 12 hypersurfaces both at $z_1 = z_2 = 0$ have a curve C of singularities blowing up of which leads to exceptional divisor E in M. The second divisor class L of both models is $K3$ fiber of fibration of M over P^1, generated by a linear system $|L|$ which is generated by the degree one polynomials. The divisor classes E and L together generate $H^4(M,Z)$. In both models the degree two polynomials generate the linear system $|H| = |2L+E|$. The homology group $H_2(M,Z)$ is generated by the classes l and h, which are defined by $h = \frac{1}{4}H \cdot L$, $l = \frac{1}{4}H \cdot E$ for degree 8 model and $h = \frac{1}{2}H \cdot L$, $l = \frac{1}{2}H \cdot E$ for degree 12 model. The intersection relations for these models are

$$\begin{aligned}
&L^3 = 0, \qquad E \cdot h = 1,\\
&E^3 = 9, \qquad E^2 \cdot L = -3, \qquad E \cdot L^2 = 1,\\
&h = L^2, \qquad l = E \cdot H \cdot L, \qquad E \cdot l = -2,
\end{aligned} \tag{19}$$

where $H = E + 3L$, for $P^4_{(1,1,1,6,9)}[18]$ hypersurface,

$$\begin{aligned}
&H^3 = 8, \qquad H^2 \cdot L = 4, \qquad H \cdot L^2 = 0, \qquad L^3 = 0,\\
&E^3 = 16, \qquad E^2 \cdot L = 4, \qquad E \cdot L^2 = 0, \qquad H \cdot E \cdot L = 4,\\
&h = 1/4H \cdot L, \qquad l = 1/4H \cdot E, \qquad H \cdot h = 1, \qquad E \cdot l = -2,\\
&L \cdot l = 1, \qquad L \cdot h = 0, \qquad H \cdot l = 0, \qquad E \cdot h = 1,\\
&c_2(M) \cdot E = 8, \qquad c_2(M) \cdot H = 56,
\end{aligned} \tag{20}$$

where $|H| = |2L + E|$, for $P^4_{(1,1,2,2,6)}[18]$ and

$$\begin{aligned}
&H^3 = 4, \qquad H^2 \cdot L = 2, \qquad H \cdot L^2 = 0, \qquad L^3 = 0,\\
&E^3 = -8, \qquad E^2 \cdot L = 2, \qquad E \cdot L^2 = 0, \qquad H \cdot E \cdot L = 2,\\
&h = 1/4H \cdot L, \qquad l = 1/4H \cdot E, \qquad H \cdot h = 1, \qquad E \cdot l = -2,\\
&L \cdot l = 1, \qquad L \cdot h = 0, \qquad H \cdot l = 0, \qquad E \cdot h = 1,\\
&c_2(M) \cdot E = 8, \qquad c_2(M) \cdot H = 56,
\end{aligned} \tag{21}$$

where $|H| = |2L + E|$, for $P^4_{(1,1,2,6,9)}[18]$ models. The choice (E,L) as generators of the complexified Kähler cone leads to generic Kähler class $K = t_1E + t_2L$, where (t_1,t_2) are classical coordinates on the Kähler moduli space of M.

4 D-branes and periods

In the points of large volume limit of Kähler moduli space we can relate the BPS charge lattice of the low energy effective theory (the integral symplectic lattice $H^3(W,Z)$ of the middle cohomology of mirror manifold W) with BPS charge lattice of microscopic D-brane charges (the integer quadratic lattice $K(M)$ of K-theory of manifold M). An every vector $\boldsymbol{n} = (n_6, n_4^1, n_4^2, n_0, n_2^1, n_2^2)$ of middle cohomology $H^3(W, Z)$ of the mirror manifold W defines central charge

$$Z(\boldsymbol{n}) = n_6\Pi_1 + n_4^1\Pi_2 + n_4^2\Pi_3 + n_0\Pi_4 + n_2^1\Pi_5 + n_2^2\Pi_6. \tag{22}$$

On the other hand every element $\eta \in K(M)$ corresponding to topological invariant $\mathrm{ch}\,(\eta)$ defines the effective charge (the Mukai vector) of D-brane

$$Q = \mathrm{ch}\,(\eta)\sqrt{\mathrm{Td}\,(\mathcal{E})} \in H^0(M) \oplus H^2(M) \oplus H^4(M) \oplus H^6(M). \tag{23}$$

The central charge associated to the state η is

$$Z(t) = \int_M \frac{t^3}{6}Q^0 - \frac{t^2}{2}Q^2 + tQ^4 - Q^6. \tag{24}$$

Equating (22) and (24) we obtain relations between the topological invariant of the K-theory class η and low energy charges of a D-brane $\boldsymbol{n} = (n_6, n_4^1, n_4^2, n_0, n_2^1, n_2^2)$ in the large radius limit.

5 Examples

For example, in type IIB superstring theory we consider the topological invariants of D6-branes wrapped on $P^4_{(1,1,1,6,9)}[18]$.

In this case $\mathcal{E}$ is the vector bundle (the coherent sheaf) on the manifold M. The standard formulae

$$\mathrm{ch}\,(\mathcal{E}) = r + c_1 + \frac{1}{2}\left(c_1^2 - 2c_2\right) + \frac{1}{6}\left(c_1^2 - 3c_1c_2 + c_3\right) + \cdots,$$
$$\mathrm{Td}\,(\mathcal{E}) = 1 + \frac{1}{2}c_1 + \frac{1}{24}c_1c_2 + \cdots.$$

give possibility to rewrite the charge (23) in the form

$$Q = \left(r, c_1(\mathcal{E}), \mathrm{ch}_2(\mathcal{E}) + \frac{r}{24}c_2(M), \mathrm{ch}_3(\mathcal{E}) + \frac{1}{24}c_1(\mathcal{E})c_2(M)\right). \tag{25}$$

At the large radius limit we have following expression for these periods of holomorphic three form $\hat{\Omega}$ of the mirror manifolds W.

$$\begin{pmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \\ \Pi_4 \\ \Pi_5 \\ \Pi_6 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(3t_1^3 + 3t_1^2t_2 + t_1t_2^2) + \frac{17}{4}t_1 + \frac{3}{2}t_2, \\ -\frac{1}{2}t_2^2 + \frac{3}{2}t_1 + \frac{3}{2}, \\ -\frac{1}{2}(3t_1^2 + 2t_1t_2) + \frac{3}{2}t_1 + \frac{3}{2}, \\ 1 \\ t_1 \\ t_2 \end{pmatrix}, \tag{26}$$

for $P^4_{(1,1,2,6,9)}[18]$ surface,

$$\begin{pmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \\ \Pi_4 \\ \Pi_5 \\ \Pi_6 \end{pmatrix} = \begin{pmatrix} \frac{2}{3}t_1^3 + t_1^2t_2 + \frac{13}{6}t_1 + t_2 \\ \frac{1}{6} - 2t_1t_2 \\ 1 - t_1^2 \\ 1 \\ t_1 \\ t_2 \end{pmatrix}, \quad \begin{pmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \\ \Pi_4 \\ \Pi_5 \\ \Pi_6 \end{pmatrix} = \begin{pmatrix} \frac{4}{3}(3t_1^3 + 2t_1t_2 + \frac{7}{3}t_1 + t_2 \\ -4t_1t_2 + 4t_1 - 2t_2 + \frac{1}{3} \\ -2t_1^2 - 2t_1 + 1 \\ 1 \\ t_1 \\ t_2 \end{pmatrix}. \tag{27}$$

for $P^4_{(1,1,2,2,2)}[8]$ and $P^4_{(1,1,2,2,6)}[12]$ surfaces, respectively. Using formulae (22), (23), (24), (26), (27) we obtain the topological invariants of D6-branes

$$\begin{aligned}
&r = n_6,\\
&\mathrm{ch}_1(\mathcal{E}) = n_4^1 E + n_4^2 L,\\
&\mathrm{ch}_2(\mathcal{E}) = \left(\frac{3}{2}n_4^2 + n_2^1\right) h + \left(\frac{3}{2}n_4^1 + n_2^2\right) l\\
&\mathrm{ch}_3(\mathcal{E}) = -n_0 + \frac{1}{2}n_4^1 - 3n_4^2,
\end{aligned} \tag{28}$$

wrapped on $P^4_{(1,1,1,6,9)}[18]$ surface,

$$\begin{aligned}
&r = n_6,\\
&\mathrm{ch}_1(\mathcal{E}) = n_4^1 E + n_4^2 L,\\
&\mathrm{ch}_2(\mathcal{E}) = (4n_4^1 - 2n_4^2 + n_2^1)h + (-2n_4^1 + n_2^2)l,\\
&\mathrm{ch}_2(\mathcal{E}) = -n_0 - \frac{2}{3}n_4^1 - 2n_4^2,
\end{aligned} \tag{29}$$

wrapped on $P^4_{(1,1,2,2,2)}[8]$,

$$\begin{aligned}
&r = n_6,\\
&\mathrm{ch}_1(\mathcal{E}) = n_4^1 E + n_4^2 L,\\
&\mathrm{ch}_2(\mathcal{E}) = 4n_2^1 h + n_2^2 l,\\
&\mathrm{ch}_2(\mathcal{E}) = -n_0 - \frac{1}{3}n_4^1 - 2n_4^2,
\end{aligned} \tag{30}$$

wrapped on $P^4_{(1,1,2,2,6)}[12]$.

6 Localized D-branes on hypersurfaces

Let $i : D \hookrightarrow M$ be embedding of an even projective surface D in a Calabi–Yau manifold M. We can obtain class of D-brane states by wrapping D-branes on submanifold D. Multiple brane configurations are described by coherent sheaf $\mathcal{E}$ on D. The charge of this configuration is defined by torsion coherent sheaf $i_*\mathcal{E}$ which is the extension $\mathcal{E}$ by zero to the manifold M. The Grothendieck–Riemann–Roch formula for the embedding $i : D \hookrightarrow M$ gives Mukai vector

$$i_*(\mathrm{ch}\,(\mathcal{E})\,\mathrm{Td}\,(D)) = \mathrm{ch}\,(i_*\mathcal{E})\,\mathrm{Td}\,(M).$$

From this formula and from the topological invariants of the sheaf $\mathcal{E}$ on D we obtain topological invariants of the torsion sheaf $i_*\mathcal{E}$

$$\begin{aligned}
&\mathrm{ch}_1(i_*\mathcal{E}) = rD,\\
&\mathrm{ch}_2(i_*\mathcal{E}) = i_*c_1(\mathcal{E}) + \frac{r}{2}i_*c_1(\mathcal{E}),\\
&\mathrm{ch}_2(i_*\mathcal{E}) = i_*\left(\mathrm{ch}_2(\mathcal{E}) + \frac{1}{2}c_1(\mathcal{E})c_1(D) + \frac{1}{12}(c_1(D))^2 + c_2(D)\right) - \frac{r}{12}Dc_2(M).
\end{aligned} \tag{31}$$

and Mukai charge vector

$$\begin{aligned}
Q = \Big(&0, rD, i_*c_1(\mathcal{E}) + \frac{r}{2}i_*c_1(D),\\
&\mathrm{ch}_2(\mathcal{E}) + \frac{1}{2}c_1(\mathcal{E})c_1(D) + \frac{r}{12}\big(c_1(D) + c_2(D)\big) - \frac{r}{24}Dc_2(D)\Big).
\end{aligned} \tag{32}$$

By means of an adjunction formula

$$Dc_2(M) = c_2(D) - D^2 = c_2(D) - D^3 = c_2(D) - c_1(D)^2$$

this Mukai charge vector can be represented as

$$Q = \left(0, rD, i_* c_1(\mathcal{E}) + \frac{r}{2} i_* c_1(D), \mathrm{ch}_2(\mathcal{E}) + \frac{1}{2} c_1(\mathcal{E}) c_1(D) + \frac{r}{8} c_1(D)^2 + \frac{r}{24} Dc_2(D)\right) \tag{33}$$

and central charge associated to it can be rewritten

$$\begin{aligned} Z(Q) = &-\frac{r}{2} t^2 D + \left(i_* c_1(\mathcal{E}) + \frac{r}{2} i_* c_1(D)\right) t - \mathrm{ch}_2(\mathcal{E}) \\ &- \frac{1}{2} c_1(\mathcal{E}) c_1(D) - \frac{r}{8} c_1(D)^2 - \frac{r}{24} c_1(D). \end{aligned} \tag{34}$$

In type IIB superstring theory the topological invariants D-4 branes wrapped on some hypersurfaces of the Calabi–Yau manifold can be defined. It is easy to obtain topological invariants of the D4-branes wrapped on the exceptional divisor E corresponding to BPS state with the charge vector $\boldsymbol{n} = (0, n_4^1, 0, n_0, 0, n_2^2)$ and with the central charge $Z(\boldsymbol{n}) = n_4^1 \mathcal{F}^1 + n_2^2 t^2 + n_0$ for the $P^4_{(1,1,2,6,9)}$ model

$$\begin{aligned} &r = n_4^1, \\ &c_1(\mathcal{E}) = n_l^2, \\ &c_2(\mathcal{E}) = \frac{1}{2} n_2^2 (n_2^2 + 3) + n_4^1 + n_0), \end{aligned} \tag{35}$$

and with the charge $\boldsymbol{n} = (0, n_4^1, 0, n_0, n_2^1, n_2^2)$ and the central charges $Z(\boldsymbol{n}) = n_4^1 \mathcal{F}^1 + n_2^1 t^1 + n_2^2 t_2 + n_0$ for the $P^4_{(1,1,2,2,2)}$[8] and $P^4_{(1,1,2,2,6)}$[12] models

$$\begin{aligned} &r(\mathcal{E}) = n_4^1, \\ &c_1(\mathcal{E}) = (n_2^1 + \nu n_4^1) h + \left(n_2^2 - \frac{\nu}{2} n_4^1\right) l, \\ &\mathrm{ch}(\mathcal{E}) = -\frac{3}{2} \nu n_4^1 - \frac{1}{2} n_2^1 + n_2^2 - n_0. \end{aligned} \tag{36}$$

where $\nu = -2, -4$. The same method may be used to define the topological invariants of D-branes on the surface (17). This surface is an elliptic fibration over a Hirzebruch surface which itself is a P^1-fibration over P^1. As in the cases considered above this fibration structure is essentially for investigation of the monodromy around singular points in the moduli space.

7 Monodromy

Consider complex function $f(z) = \sqrt{z}$. If we circle once the origin $z \to z \exp 2i\pi$, then $f(z \exp 2i\pi) = -f(z)$. The function does not return back to itself. This is signal that the function at the point $z = 0$ is singular and in this case it is the start of a branch cut.

The behavior of a complex function or set of functions after transport around a singular point (singularity) is connected with their monodromy. In general, a set of function transported once around the singular point t_0 return to the linear combination of themselves $f_i((z - z_0) \exp 2i\pi) = m_{ij}(z_0) f_j(z)$. The matrix m_{ij} depend on the singular point and it is called by the monodromy matrix. It does not change under smooth deformations of the contour. The monodromy has topological characteristic.

In general, one of the important problems of algebraic geometry is to study families of algebraic variety parameterized by another variety. A subvariety in moduli space parameterizing the singular fibers has the special interest. This subvariety is called the discriminant locus. One of the main its topological invariants is so-called monodromy group. It is defined by the action of homotopy group of the complement of discriminant locus on cohomology ring of fixed nonsingular fiber. The large radius limit point is situated in the intersection of the two divisors H and L on the boundary of the moduli space [6]. The monodromy matrices of the two divisors E, H, are expressed with respect to the (E, L) basis [5,6] by

$$S_L = \begin{pmatrix} 1 & -1 & -3 & 10 & 9 & 3 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -3 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \qquad S_H = \begin{pmatrix} 1 & 0 & -1 & 3 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{37}$$

for $P^4_{(1,1,1,6,9)}[18]$ model,

$$S_L = \begin{pmatrix} 1 & 0 & -1 & 2 & -2 & 0 \\ 0 & 1 & 0 & -2 & -4 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \qquad S_H = \begin{pmatrix} 1 & -1 & -2 & 6 & 4 & 0 \\ 0 & 1 & 0 & 4 & 0 & -4 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \tag{38}$$

for $P^4_{(1,1,2,2,2)}[8]$ model,

$$S_L = \begin{pmatrix} 1 & 0 & -1 & 20 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \qquad S_H = \begin{pmatrix} 1 & -1 & -2 & 5 & 2 & 1 \\ 0 & 1 & 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{39}$$

for $P^4_{(1,1,2,2,6)}[12]$ model. We can use above formulae to convert the monodromy transformations (37), (38), (39) into automorphisms of K-group $K(M)$

$$[\mathcal{E}] \to [\mathcal{E} \otimes \mathcal{O}_M(L)], [\mathcal{E}] \to [\mathcal{E} \otimes \mathcal{O}_M(H)], \tag{40}$$

where $[\mathcal{E}] \in K(M)$. The topological invariants of the sheaf $\mathcal{E}' = \mathcal{E} \otimes \mathcal{O}_M(D)$ under the transformation (40) change to the form

$$\begin{aligned} & r(\mathcal{E}') = r(\mathcal{E}), \\ & \mathrm{ch}_1(\mathcal{E}') = \mathrm{ch}_1(\mathcal{E}) + rD, \\ & \mathrm{ch}_2(\mathcal{E}') = \mathrm{ch}_2(\mathcal{E}) + \mathrm{ch}_1(\mathcal{E})D + \frac{r}{2}D^2, \\ & \mathrm{ch}_3(\mathcal{E}') = \mathrm{ch}_3(\mathcal{E}) + \mathrm{ch}_2(\mathcal{E})D + \frac{1}{2}\mathrm{ch}_1(\mathcal{E})D^2 + \frac{r}{6}D^3, \end{aligned} \tag{41}$$

where $D = H$, L. The representations of these transformations on the space of charge vectors $\boldsymbol{n} = (n_0, n_4^1, n_4^2, n_0, n_2^1, n_2^2)$ and transformations (37), (38), (39) are connected by the relation

$$M(D) = S_D^{-1}. \tag{42}$$

8 Conclusions

In this paper, we have considered the method of the D-brane constructions in superstring theory corresponding to Calabi–Yau compactification where Calabi–Yau manifolds appear as hypersurfaces in the weighted projective space. In the large volume "phase" of the Kähler moduli space this construction provides clear geometric picture of the D-brane spectrum. We find that this construction gives clear picture of the large volume monodromy transformations. All the above is a simple particular case of a more ambitious program which proposes for type IIB superstring theory an identification of the D-branes by objects of a derived category of the category of coherent sheaves [7, 8] (see also [9]).

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