

Self-Gravitating Global Monopole and Nonsingular Cosmology

K.A. BRONNIKOV[†] and *B.E. MEIEROVICH*[‡]

[†] *VNIIMS, 3-1 M. Ulyanovoy Str., 117313 Moscow, Russia;
Institute of Gravitation and Cosmology, PFUR,
6 Miklukho-Maklaya Str., 117198 Moscow, Russia
E-mail: kb@rgs.mccme.ru*

[‡] *Kapitza Institute for Physical Problems, 2 Kosygina Str., 117334 Moscow, Russia
E-mail: meierovich@yahoo.com*

We review some recent results concerning the properties of a spherically symmetric global monopole in $(D = d + 2)$ -dimensional general relativity. Some common features of monopole solutions are found independently of the choice of the symmetry-breaking potential. Thus, the solutions show six types of qualitative behavior and can contain at most one simple horizon. For the standard Mexican hat (Higgs) potential, we analytically find the D -dependent range of the parameter γ (characterizing the gravitational field strength) in which there exist globally regular solutions with a monotonically growing Higgs field, containing a horizon and a Kantowski–Sachs (KS) cosmology outside it, with the topology of spatial sections $\mathbb{R} \times \mathbb{S}^d$. Their cosmological properties favor the idea that the standard Big Bang might be replaced with a nonsingular static core and a horizon appearing as a result of some symmetry-breaking phase transition on the Planck energy scale. We have also found families of new solutions with an oscillating Higgs field, parametrized by the number of its knots. All such solutions describe space-times of finite size, possessing a regular center, a horizon and a singularity beyond it.

According to the Standard cosmological model, the Universe has been expanding and cooling from a split second after the Big Bang to the present time and remained uniform and isotropic on the large in doing so. In the process of its evolution, the Universe has experienced a chain of phase transitions with spontaneous symmetry breaking, including Grand Unification and electroweak phase transitions, formation of neutrons and protons from quarks, recombination, and so forth. Regions with spontaneously broken symmetry which are more than the correlation length apart, are statistically independent. At interfaces between these regions, the so-called topological defects necessarily arise.

A systematic exposition of the potential role of topological defects in our Universe has been provided by Vilenkin and Shellard [1]; see there also the necessary references to the previous work. The particular types of defects: domain walls, strings, monopoles, or textures are determined by the topological properties of vacuum. If the vacuum manifold after the breakdown is not shrinkable to a point, then solutions of Polyakov–’t Hooft monopole type appear in quantum field theory.

Spontaneous symmetry breaking is well known to play a fundamental role in modern attempts to construct particle theories. In this context, one mostly deals with internal symmetries rather than with those associated with space-time transformations: examples are the Grand Unification symmetry, the electroweak and isotopic symmetries and supersymmetry, whose transformations mix bosons and fermions. Topological defects, caused by spontaneous breaking of internal symmetries independent of space-time coordinates, are called global.

A fundamental property of global symmetry violation is the Goldstone degree of freedom. In the monopole case, the term related to the Goldstone boson in the energy-momentum tensor

decreases rather slowly away from the center. As a result, the total energy of a global monopole grows linearly with distance, in other words, diverges. Without gravity such a divergence is a general property of spontaneously broken global symmetries. The self-gravity of a global monopole, if not entirely removes this difficulty, allows one to consider it from a new standpoint.

We have performed a general study of the properties of static, spherically symmetric global monopoles in general relativity [2]. Most of the results were extended [3] to spherically symmetric configurations of arbitrary dimension with the topology $\mathbb{R} \times \mathbb{R}_+ \times \mathbb{S}^d$, but in this presentation we shall for simplicity mostly adhere to $D = 4$ ($d = 2$). In this case, the Lagrangian is taken in the form

$$L = \frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a - V(\phi) + \frac{R}{16\pi G}, \quad (1)$$

where R is the scalar curvature, G is the gravitational constant, ϕ^a , $a = 1, 2, 3$ is a scalar field multiplet, and $V(\phi)$ is a symmetry-breaking potential depending on $\phi = \pm \sqrt{\phi^a \phi^a}$. We assume the static, spherically symmetric metric

$$ds^2 = A(\rho) dt^2 - \frac{d\rho^2}{A(\rho)} - r^2(\rho) d\Omega^2 \quad (2)$$

($d\Omega^2 \equiv d\theta^2 + \sin^2 \theta d\varphi^2$) and a ‘‘hedgehog’’ scalar field configuration:

$$\phi^a = \phi(\rho) n^a, \quad n^a = \{\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta\}. \quad (3)$$

Without loss of generality we take $\rho \geq 0$ and attribute $\rho = 0$ to a regular center.

Our approach was different from most of previous studies which had used the boundary condition that $V = 0$ at spatial infinity. We did not even require the existence of a spatial asymptotic. Instead, we required regularity at the center and tried to observe the properties of the whole set of global monopole solutions. Also, instead of dealing only with a particular form of the symmetry breaking potential (usually the Mexican hat potential), we found some general features of solutions valid independently of the particular shape of $V(\phi)$. The main results are as follows.

The Einstein equations lead to $r'' \leq 0$. Since at a regular center $r' > 0$, this leaves three possibilities for the function $r(\rho)$:

- (a) monotonic growth with a decreasing slope, but $r \rightarrow \infty$ as $\rho \rightarrow \infty$,
- (b) monotonic growth with $r \rightarrow r_{\max} < \infty$ as $\rho \rightarrow \infty$, and
- (c) growth up to r_{\max} at some $\rho_1 < \infty$ and further decrease, reaching $r = 0$ at some finite $\rho_2 > \rho_1$.

The other metric function, $A(\rho)$, determines the causal structure of space-time: zeros of any order of $A(\rho)$ correspond to Killing horizon of the same order.

Proposition 1. *Under the assumption that $\phi^2 < 1/(8\pi G)$ in the whole space, our system with a regular center can have either no horizon, or one simple horizon, and in the latter case its global structure is the same as that of de Sitter space-time.*

For nonnegative $V(\phi)$ there is no restriction on the magnitude of ϕ .

Proposition 2. *If $V(\phi) \geq 0$, our system with a regular center can have either no horizon, or one simple horizon, and in the latter case its global structure is the same as that of de Sitter space-time.*

Outside the horizon, in the so-called T-region, ρ becomes a temporal coordinate, and the geometry corresponds to homogeneous anisotropic cosmological models of Kantowski–Sachs (KS) type, where spatial sections have the topology $\mathbb{R} \times \mathbb{S}^2$.

Thus, depending on the behavior of $r(\rho)$ (items a–c) and $A(\rho)$ (with or without a horizon), all possible solutions can be divided into six qualitatively different classes. There are two more general results valid for any nonnegative potentials.

Proposition 3. *If $V(\phi) \geq 0$, the second center $r = 0$, if any, is singular.*

Proposition 4. *If $V(\phi) \geq 0$ and the solution is asymptotically flat, the mass M of the global monopole is negative.*

In Proposition 4, asymptotic flatness is understood up to the solid angle deficit $\Delta < 1$: at large ρ we have $r(\rho) \approx \rho$ and

$$ds^2 = \left(\frac{1 - \Delta}{\alpha^2} - \frac{2GM}{r} \right) dt^2 - \left(\frac{1 - \Delta}{\alpha^2} - \frac{2GM}{r} \right)^{-1} dr^2 - r^2 d\Omega^2, \quad (4)$$

where α is a model-dependent constant. To our knowledge, Proposition 4 had been obtained previously only numerically for the particular potential (5) (see below).

Thus, even before studying particular solutions, we have a more or less complete knowledge of what can be expected from such global monopole systems.

We further considered [2] the most frequently used Mexican hat potential

$$V(\phi) = \frac{1}{4} \lambda (\phi^a \phi^a - \eta^2)^2 = \frac{1}{4} \eta^4 \lambda (f^2 - 1)^2, \quad (5)$$

where $\eta > 0$ characterizes the energy of symmetry breaking, λ is a dimensionless coupling constant and $f(\rho) = \phi(\rho)/\eta$ is the normalized field magnitude playing the role of an order parameter. The model has a global $SO(3)$ symmetry, which can be spontaneously broken to $SO(2)$ due to the potential wells ($V = 0$) at $f = \pm 1$.

Our analytical and numerical study for the potential (5) has confirmed previous results of other authors concerning configurations with a monotonically growing field magnitude f .

The solution properties are basically governed by the values of the single dimensionless parameter

$$\gamma = 8\pi G\eta^2, \quad (6)$$

characterizing the gravitational field strength. Thus, for $\gamma < 1$ the solutions have a spatial asymptotic with the metric (4). For $1 < \gamma < 3$, the so-called supermassive global monopole, the solutions contain a cosmological horizon and a KS model outside it.

We have obtained analytically the upper limit $\gamma_0 = 3$ for the existence of static monopole solutions, previously found numerically by Liebling [4]. To do so, we have used the fact that near a critical value of γ the field magnitude is small everywhere inside the horizon, making it possible to formulate a well-posed eigenvalue problem for the field f against the background of the de Sitter metric (which solves the Einstein equations in case $f \equiv 0$). The linear equation for f has the form

$$\frac{d}{dx} \left[x^2 \left(1 - \frac{x^2}{x_h^2} \right) \frac{df}{dx} \right] - (2 - x^2) f = 0, \quad (7)$$

where x is a dimensionless variable proportional to r and $x_h = \sqrt{12/\gamma}$ is the value of x at the horizon. The boundary conditions are

$$f \Big|_{x=0} = 0, \quad |f(x_h)| < \infty. \quad (8)$$

Nontrivial solutions of (7) with these boundary conditions exist for the sequence of eigenvalues

$$\gamma_n = \frac{3}{(n + 1/2)(n + 2)}, \quad n = 0, 1, 2, \dots, \quad (9)$$

where n is the number of nodes of the corresponding eigenfunctions $f_n(x)$. The eigenvalue $\gamma_0 = 3$ is the sought-for critical value of γ for monotonically growing f . In case $\gamma > 3$ static monopole solutions are absent.

The solutions with $n \geq 1$ form new families, which we had not met in the existing literature. In these solutions, existing for $\gamma < \gamma_n$, the field function f (which has no reason to be small when γ is far from γ_n), changes its sign n times. All such solutions turn out to have a singularity ($f \rightarrow \infty$, $r \rightarrow 0$) at some finite value of ρ beyond the horizon.

The solutions with a static nonsingular monopole core and a KS cosmological model in a T-region ($A(\rho) < 0$) outside the horizon are of particular interest. Changing the notations, $t \rightarrow y \in \mathbb{R}$, and introducing the proper time of a comoving observer $\tau = \int d\rho/\sqrt{|A(\rho)|}$, we can rewrite the metric as

$$ds^2 = d\tau^2 - |A(\tau)|dy^2 - r^2(\tau)d\Omega^2. \quad (10)$$

The model expands in one of the directions (along the y axis) from zero at the horizon (say, $\tau = 0$) to finite values at large τ in a process which, on its early stage, resembles inflation. In two other directions, corresponding to \mathbb{S}^2 , the model expands from a finite size and finite expansion rate at $\tau = 0$ to a linear regime at large τ . Like other regular models with the de Sitter causal structure, i.e., a static core and expansion beyond a horizon, these models drastically differ from standard Big Bang models in that the expansion starts from a nonsingular surface, and cosmological comoving observers can receive information in the form of particles and light quanta from the static region, situated in the absolute past with respect to them. Moreover, in our case the static core is nonsingular, and it is thus an example of an entirely nonsingular cosmology in the spirit of the views advocated by Gliner and Dymnikova [5, 6] (see also references therein).

The nonzero symmetry-breaking potential plays the role of a time-dependent cosmological constant, a kind of hidden vacuum matter. For an observer in the T-region the potential decreases with time, and the hidden vacuum matter gradually disappears.

The present simple model cannot be directly applied to our Universe (in particular, due to lack of isotropization), it can at most pretend to describe the earliest, near-Planckian stage in an approximate, classical manner. It nevertheless may be considered as an argument in favor of the idea that the standard Big Bang might be replaced with a nonsingular static core and a horizon appearing as a result of some symmetry-breaking phase transition on the Planck energy scale.

As another cosmological application of the global monopole, one should mention the concept of topological inflation, related to the existence of a de Sitter core of the monopole, which can inflate due to its instability [7].

In Ref. [3] we have extended the above consideration to $(D = d + 2)$ -dimensional general relativity, with the space-time topology $\mathbb{R} \times \mathbb{R}_+ \times \mathbb{S}^d$. The qualitative features of the solutions are mostly preserved, in particular, there are the same six types of behavior, and Propositions 1–4 still hold. Outside the horizon (if any), the metric again corresponds to a Kantowski–Sachs type cosmology, now with the topology of spatial sections $\mathbb{R} \times \mathbb{S}^d$.

For the Mexican hat potential (5), the strength parameter is defined as $\gamma = \kappa^2 \eta^2$, where κ^2 is the gravitational constant of D -dimensional theory and η is the symmetry breaking characteristic from (5). There are again two critical values of γ for each D : for monotonic $f(\rho)$, solutions with static spatial infinity exist with $\gamma < d - 1$, while solutions with a horizon and an infinitely expanding KS exterior correspond to $d - 1 < \gamma < 2d(d + 1)/(d + 2)$. For solutions where $f(\rho)$

changes its sign n times, instead of (9), the critical values of γ are

$$\gamma_n = \frac{2d(d+1)}{(2n+1)(2n+d+2)}, \quad (11)$$

which reduces to (9) when $d = 2$.

In the important case when the horizon is far from the monopole core, the temporal evolution of the KS metric is described analytically. The Kantowski–Sachs space-time contains a subspace with a closed Friedmann–Robertson–Walker metric. Our estimates show that the 5-dimensional global monopole model is in principle able to give plausible cosmological parameters. However, within our macroscopic theory without specifying the physical nature of vacuum we cannot unequivocally explain why the fourth spatial dimension (the one that played the role of time in the static region) is not observable. Quantitative estimates certainly require a more complete model including further phase transitions, one of which should explain the unobservable nature of the extra dimension.

- [1] Vilenkin A. and Shellard E.P.S., *Cosmic strings and other topological defects*, Cambridge, Cambridge University Press, 1994.
- [2] Bronnikov K.A., Meierovich B.E. and Podolyak E.R., Global monopole in general relativity, *Zh. Eksp. Teor. Fiz.*, 2002, V.122, N 3, 459–471 (translated in *JETP*, 2002, V.95, N 3, 392–403).
- [3] Bronnikov K.A. and Meierovich B.E., Multidimensional global monopole and nonsingular cosmology, *Zh. Eksp. Teor. Fiz.*, 2002, V.124, N 1, 5–13 (translated in *JETP*, 2003, V.97, N 1, 1–9).
- [4] Liebling, S.L., Static gravitational global monopoles, *Phys. Rev. D*, 2000, V.61, 024030.
- [5] Gliner E.B., Inflationary universe and the vacuumlike state of physical medium, *Usp. Fiz. Nauk*, 2002, V.45, 221–227;
Gliner E.B. and Dymnikova I.G., Nonsingular Freidmann cosmology, *Pis'ma v Astron. Zh.*, 1975, V.1, 7 (reprinted in *Usp. Fiz. Nauk*, 2002, V.45, 227–228).
- [6] Dymnikova I., Cosmological term, mass, and space-time symmetry, hep-th/0310047;
Bronnikov K.A., Dobosz A. and Dymnikova I.G., Nonsingular vacuum cosmologies with a variable cosmological term, *Class. Quant. Grav.*, 2003, V.20, 3797–3814.
- [7] Cho Inyong and Vilenkin A., Spacetime structure of an inflating global monopole, *Phys. Rev. D*, 1997, V.56, 7621–7626.