The Multiple Point Principle: Realized Vacuum in Nature is Maximally Degenerate

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We put forward the multiple point principle as a fine-tuning mechanism that can explain why the parameters of the standard model have the values observed experimentally. The principle states that the parameter values realized in Nature coincide with the surface (e.g. the point) in the action parameter space that lies in the boundary that separates the maximum number of regulator-induced phases (e.g., the lattice artifact phases of a lattice gauge theory). We argue that a mild form of non-locality – namely that embodied in allowing diffeomorphism invariant contributions to the action – seems to appear in any fine-tuning problem. We demonstrate that the multiple point principle solution to fine-tuning has the very special property of avoiding the paradoxes that can arise in the presence of non-locality.

1 Introduction

The Multiple Point Principle (MPP) states that fundamental physical parameters assume values that correspond to having a maximal number of different coexisting "phases" for the physically realized vacuum. There is phenomenological evidence suggesting that some or all of the about 20 parameters in the Standard Model (SM) that are not predicted within the framework of the SM correspond to the MPP values of these parameters [1–3]. That these parameters take on special values (i.e., the multiple point values) poses from one viewpoint a fine tuning problem (why do constants of Nature take the MPP values). From another viewpoint, assuming the MPP as a law of Nature leads to a mechanism for fine-tuning. It is the latter viewpoint that is developed here. Moreover, we shall argue that a mild form of non locality is inherent to fine-tuning problems in general. We therefore develop a model for the relationship between fine-tuning, non-locality and the MPP.

2 Fine-tuning seems to require some form of non-locality

Explaining the (dressed) value of the cosmological constant is an example of a fine-tuning problem that would seem to require the breakdown of locality at least in a mild sense. As with any fine-tuning problem, the cosmological constant problem calls for a way to make the coupling dynamical in such a way that the values of such couplings are maintained at constant values (required for translational invariance). But this leads to a problem: if a coupling (e.g., the cosmological constant) is dynamical, the demands of a strictly local theory would be that the bare coupling can only depend on the space-time point in question and indirectly on the past but certainly not on the future. However, if the bare cosmological constant (that is to be dynamically maintained at a constant value) immediately following the big bang is to already have its value fine-tuned once and for all – to say 120 decimal places – to the value that makes the dressed cosmological constant so small as suggested phenomenologically, we definitely have a problem with locality.

The problem is that the bare cosmological constant is relatable to the value of the dressed cosmological constant only if the details of the dressed cosmological constant (that did not exist when the bare value was already tuned to the valued required for the dressed vacuum) that will evolve in the future are known at the time of big bang [4]. We are forced to conclude that a strict principle of locality is not allowed if we want to have a dynamically maintained bare coupling and renormalization group corrections of a quantum field theory with a well-defined vacuum.

This suggests models with a mild form of non-locality consisting of an interaction that is the same between any pair of points in space-time independent of the distance between these points. Assuming that this sort of non-locality is manifested through a non-local action $\hat{S}_{\mathrm{nl}}$, this symmetry between any pair of space-time points (i.e., identical interaction regardless of separation) is insured by requiring the invariance of $\hat{S}_{\mathrm{nl}}$ under diffeomorphisms (reparameterization invariance). The non-local action $\hat{S}_{nl}$ is a function of functionals $I_{f_j}[\phi(x)]$: $\hat{S}_{\mathrm{nl}} = \hat{S}_{\mathrm{nl}}(\{I_{f_j}[\phi(x)]\})$ where $I_{f_j}[\phi(x)] \stackrel{\text{def}}{=} \int dx^4 \sqrt{g(x)} f_j(\phi(x))$ and $f_j(\phi)$ might typically be a Lagrange density e.g., $f_j(\phi) = \mathcal{L}_j(\phi(x), \partial_\mu \phi(x))$. The symbol $\phi(x)$ stands for all the fields (and derivatives of same) of the theory.

An example of a nonlocal action would be any nonlinear function of the (reparameterization invariant) functionals $I_{f_i}, I_{f_j}, \ldots$; e.g., a term $\int d^4x \int d^4y \sqrt{g(x)g(y)} \phi^2(x)\phi^4(y)$. Another example of a non-local (and nonlinear) action term more relevant to this paper is associated with having fixed values $I_{\mathrm{fixed}\, f_j}$ (fixed in the sense of being a law of Nature) of some extensive quantities $I_{f_j}[\phi]$. This amounts to having a δ-function term $\exp(S_{\mathrm{nl}}(\{I_{f_j}\})) = \prod_j \delta(I_{f_j}[\phi] - I_{\mathrm{fixed}\, f_j})$ in the functional integration measure and results in the non-locality that, strictly speaking, is inherent to any microcanonical ensemble (but which often is "approximated away" by using a canonical ensemble when phase space volume (or functional integration measure) is a sufficiently rapidly varying function of the extensive quantities).

An extensive quantity $I_{f_j}[\phi(x)]$ has a value for each imaginable Feynman path integral history of the Universe as it evolves from Big Bang to Big Crunch. The value $I_{\mathrm{fixed}\, f_j}$ is by assumption "frozen in" and cannot change during the lifetime of the Universe. This unchangeable "choice" $I_{\mathrm{fixed}\, f_j}$ then singles out a subset of all possible Feynman path integral histories that is consistent with the space-time evolution of our actually realized Universe having $I_{f_j}[\phi] = I_{\mathrm{fixed}\, f_j}$.

An interaction that is the same between the fields at *any* pair of space-time points – regardless of separation – would not likely be perceived as a non-local interaction. Rather such space-time omnipresent fields – a sort of background that is forever everywhere the same – would likely be interpreted as simply constants of Nature. This feature is reminiscent of baby universe theory the essence of which is that a physical constant can depend on something and still be a constant as a function of space-time.

3   The Multiple Point Principle (MPP)

The MPP was originally put forward in connection with theoretical predictions for the values of the three gauge coupling constants [1, 2]. In addition to the assumption of the MPP, we also assumed in this first application of MPP our so-called Anti-GUT gauge group $G_{\mathrm{Anti-GUT}}$ which consists of the 3-fold replication of the Standard Model Group (SMG): $G_{\mathrm{Anti-GUT}} = \mathrm{SMG} \otimes \mathrm{SMG} \otimes \mathrm{SMG} \stackrel{\text{def}}{=} \mathrm{SMG}^3$ (in the extended version: $(\mathrm{SMG} \times U(1))^3$) having one SMG factor for each generation of fermions and gauge bosons. We postulate that $G_{\mathrm{Anti-GUT}}$ is broken to the diagonal subgroup (i.e., the usual SMG) at roughly the Planck scale.

In the original context of predicting the standard model gauge couplings using MPP (originally referred to as the principle of multiple point criticality), the principle asserts that the Planck scale values of the standard model gauge group couplings coincide with the multiple point, i.e., the point that lies in the boundary separating the maximum number of phases in

the action parameter space corresponding to the gauge group $G_{\text{Anti-GUT}}$. The (Planck scale) predictions for the gauge couplings are subsequently identified with the parameter values at the point in the action parameter space for the diagonal subgroup of $G_{\text{Anti-GUT}}$ that is inherited from the multiple point for $G_{\text{Anti-GUT}}$ after the Planck scale breakdown of the latter.

The idea was developed in the context of lattice gauge theory and the phases to which we refer are usually dismissed as lattice artifacts (e.g., a Higgsed phase, a confined or Coulomb-like phase). Such phases have been studied extensively in the literature for simple gauge groups and semi simple gauge groups with discrete subgroups (e.g. $SU(2)$ and $SU(3)$). One typically finds first order phase transitions between confined and Coulomb-like phases at critical values of the action parameters.

Taking such lattice artifact phases as physical reflects our suspicion that such phases are inherent to having a regulator. As a regulator in some form (be it a lattice, strings or whatever) is always needed for the consistency of any quantum field theory, it is consistent to assume the existence of a fundamental regulator. The "artifact" phases that arise in a theory with such a fundamental regulator (that we have chosen to implement as a fundamental lattice) are accordingly taken as ontological phases that have physical significance at the scale of the fundamental regulator (e.g., lattice). The assumption of an ontological fundamental regulator implies the existence of monopoles in terms of which the regulator induced phase can also be studied [5].

Finding the multiple point in an action parameter space corresponding to the gauge group $G_{\text{Anti-GUT}}$ is more complicated than for a single $SU(2)$ or $SU(3)$ say. The boundaries between phases in the action parameter space (i.e., the phase diagram) must be sought in a high dimensional parameter space essentially because $G_{\text{Anti-GUT}}$ being a non-simple group has many subgroups and invariant subgroups.

In fact, there is a distinct phase for each subgroup pair (K, H), where K is a subgroup and H is an invariant subgroup such that $H \triangleleft K \subseteq G_{\text{Anti-GUT}}$. An element $U \in G_{\text{Anti-GUT}}$ can be parameterized as $U = U(g, k, h)$ where the Higgsed (gauge) degrees of freedom are elements g of the homogeneous space $G_{\text{Anti-GUT}}/K$. The (un-Higgsed) Coulomb-like and confined degrees of freedom are respectively the elements k of the factor group K/H and the elements $h \in H$.

4   A familiar analogy to the MPP as a fine-tuning mechanism

Some important features of the MPP as a fine-tuning mechanism can be illustrated using an analogy to the familiar system in which the solid, liquid and vapor phases of water coexist. This occurs at the "triple point" of water, i.e., at the "triple point" values of temperature and pressure. Because the transitions between these three phases are all first order, there is a whole range of combinations of the extensive variables energy and volume for which the system can only be realized by having the coexistence of the ice, liquid and vapor phases. But these three phases coexist only for the triple point values of temperature and pressure, so there is a whole range of combinations of energy and volume that map onto the triple point values of the conjugate intensive variables temperature and pressure with the result that these variables are fine-tuned to the triple point values. In this illustrative analogy, the triple point of water in the phase diagram spanned by the intensive parameters temperature and pressure is analogous to the multiple point. As already stated, the multiple point is the (or a) point in the phase diagram that "touches" the maximum number of phases. In a phase diagram spanned by D intensive parameters (couplings), a generic multiple point can be in contact with up to $D + 1$ phases (in the illustrative example, $D = 2$ and the triple point is in contact with the $D + 1 = 3$ phases ice, liquid and vapour). In a non-generic situation, the multiple point can be in contact with more than $D + 1$ phases (e.g., accidently or due to symmetries).

For ease of illustration, consider now an even simpler system consisting of n_{H_2O} moles of H_2O in which just the ice and liquid phases coexist (at constant pressure). Such a system is unavoidably realized (and the temperature fine-tuned to $0°$ C $= 273.15°$ K) for *any* value of the energy density $\rho_E = E/V_{n_{H_2O}}$ (E and $V_{n_{H_2O}}$ are respectively the energy and volume of the n_{H_2O} moles of H_2O) in the *finite* interval

$$\frac{n_{H_2O}}{V_{n_{H_2O}}} \int_{0° \text{ K}}^{273° \text{ K}} C_{p,\text{ice}}(T)dT < \rho_E < \frac{n_{H_2O}}{V_{n_{H_2O}}} \left(\int_{0° \text{ K}}^{273° \text{ K}} C_{p,\text{ice}}(T)dT + (\text{molar heat of melting}) \right)$$

($C_{p,\text{ice}}$ is the molar heat capacity of ice at constant pressure (e.g., 1 atm.)).   (1)

For any ρ_E in this interval, the system cannot be realized as a single phase but rather only as an equilibriated mixture of ice and liquid water. Even choosing ρ_E at random there is a finite chance of landing in this interval in which case the temperature will be fine-tuned to $273.15°$ K.

5   The history of our universe as a fine tuner

Consider an analogy between the (3-dimensional) ice-water system with ρ_E in the interval of equation (1) and our 4-dimensional universe with the value of an extensive variable $I_{f_j}[\phi(x)] \stackrel{\text{def}}{=} \int dx^4 \sqrt{g(x)} f_j(\phi(x))$ (with f_j any function of ϕ – see also Section 2 for notation) primordially fixed at a value $I_{\text{fixed } f_j}$ that can only be realized as a combination of two (for the sake of example – really there could be more than two) coexisting phases i.e., two degenerate vacuum states at field values that we denote as ϕ_{us} and ϕ_{other} where we take $\phi_{\text{us}} < \phi_{\text{other}}$. Here we are anticipating the introduction of an effective potential V_{eff} that has relative minima at the field values ϕ_{us} and ϕ_{other}. In 4-space, one generic possibility for having coexistent phases would be to have a phase with ϕ_{us} in an early epoch including, say, the universe as we know it and a phase with ϕ_{other} in a later epoch:

$$I_{\text{fixed } f_j} = f_j(\phi_{\text{us}})(t_{\text{ignit}} - t_{BB})V_3 + f_j(\phi_{\text{other}})(t_{BC} - t_{\text{ignit}})V_3, \tag{2}$$

where t_{ignit} is the "ignition" time (in the future) at which there is a first-order phase transition from the vacuum at ϕ_{us} to the later vacuum at ϕ_{other}. V_3 is the 3-volume of the universe. The value of the "coupling constant" conjugate to $I_{\text{fixed } f_j}$ gets fine-tuned (unavoidably by assumption of the coexistence of the two phases separated by a first order transition) by a mechanism that also depends on a phase that will first be realized in the future (at t_{ignit}). Such a mechanism is non-local. Note, in particular, that the right-hand side of equation (2) depends on t_{ignit}.

In order to formally define a "coupling constant" (intensive quantity) conjugate to some extensive quantity (e.g., $I_{\text{fixed } f_j}$), we introduce non-locality more abstractly. Let us restrict the non-local action $\hat{S}_{\text{nl}} = \hat{S}_{\text{nl}}(\{I_j[\phi(x)]\})$ to being a (also reparameterization invariant) non-local potential V_{nl} that is a function of (not necessarily independent) functionals

$$V_{\text{nl}} \stackrel{\text{def}}{=} V_{\text{nl}}(I_{f_i}[\phi], I_{f_j}[\phi], \ldots).$$

Define now an effective potential V_{eff} such that

$$\frac{\partial V_{\text{eff}}(\phi(x))}{\partial \phi(x)} \stackrel{\text{def}}{=} \frac{\delta V_{\text{nl}}(\{I_{f_j}[\phi]\})}{\delta \phi(x)} \bigg|_{\text{near min}} = \sum_i \left(\frac{\partial V_{\text{nl}}(\{I_{f_j}\})}{\partial I_{f_i}} \frac{\delta I_{f_i}[\phi]}{\delta \phi(x)} \right) \bigg|_{\text{near min}}$$

$$= \sum_i \frac{\partial V_{nl}(\{I_{f_j}\})}{\partial I_{f_i}} \bigg|_{\text{near min}} f_i'(\phi(x)). \tag{3}$$

The subscript "near min" denotes the approximate ground state of the whole universe, up to deviations of $\phi(x)$ from its vacuum value (or vacuum values for a multi-phase vacuum) by any amount in relatively small space-time regions. The solution to equation (3) is

$$V_{\text{eff}}(\phi) = \sum_i \frac{\partial V_{\text{nl}}(\{I_{f_j}\})}{\partial I_{f_i}} f_i(\phi). \tag{4}$$

We can identify the $\frac{\partial V_{\text{nl}}(\{I_{f_j}\})}{\partial I_{f_i}}$ as intensive quantities conjugate to the I_{f_i}.

Consider now the effective potential (4) in the special case that $V_{\text{nl}}(\{I_{f_j}\}) = V_{\text{nl}}(I_2, I_4) \overset{\text{def}}{=}$ $V_{\text{nl}}(\int d^4x \sqrt{g(x)}\phi^2(x), \int d^4y \sqrt{g(y)}\phi^4(y))$ in which case, (4) becomes

$$V_{\text{eff}} = \frac{\partial V_{\text{nl}}(I_2, I_4)}{\partial I_2}\phi^2(x) + \frac{\partial V_{\text{nl}}(I_2, I_4)}{\partial I_4}\phi^4(x) \overset{\text{def}}{=} \frac{1}{2}m_{\text{Higgs}}^2\phi^2(x) + \frac{1}{4}\lambda\phi^4(x), \tag{5}$$

where the right-hand side of this equation, which also defines the (intensive) couplings m_{Higgs}^2 and λ, is recognised as a prototype scalar potential at the tree level. Of course, the form of V_{nl} is, at least *a priori*, completely unknown to us, so – for example – the coupling constant m_{Higgs}^2 cannot be calculated from equation (5). The potential of equation (5) with $m_{\text{Higgs}}^2 < 0$ has an asymmetric minimum – at, say, the value ϕ_{us} resulting in spontaneous symmetry breakdown in the familiar way. This is just standard physics (without non-locality).

Actually, we want to consider the potential V_{eff} having the two relative minima ϕ_{us} and ϕ_{other} – both at nonvanishing values of ϕ – alluded to at the beginning of this section. The second minimum comes about at a value $\phi_{\text{other}} > \phi_{\text{us}}$ when radiative corrections to (5) are taken into account and the top quark mass is not too large [6, 7, 3]. Which of these vacua – the one at ϕ_{us} or ϕ_{other} – would be the stable one in this two-minima Standard Model effective Higgs field potential, depends on the value of m_{Higgs}^2. Since I_2 and I_4 are functions of t_{ignit} (as seen from equation (2) with $f_j = \phi^2$ or ϕ^4), $m_{\text{Higgs}}^2 \overset{\text{def}}{=} \frac{\partial V_{\text{nl}}(\{I_2,I_4\})}{\partial I_2}$ is also a function of t_{ignit}.

Let us first use "normal physics" to see how the relative depths of the two minima of the double well are related to m_{Higgs}^2 and to t_{ignit}. It can be deduced from [7] that a large negative value of m_{Higgs}^2 corresponds to the relative minimum $V_{\text{eff}}(\phi_{\text{other}})$ being deeper than $V_{\text{eff}}(\phi_{\text{us}})$ (in which by assumption the Universe starts off following Big Bang) than for smaller negative values of m_{Higgs}^2 (see Fig. 1). It can also be argued quite plausibly that a minimum in V_{eff} at ϕ_{other} much deeper than that at ϕ_{us} would correspond to an early (small) t_{ignit} inasmuch as the "false" vacuum at ϕ_{us} would be very unstable. However, as the value of the potential at ϕ_{other} approaches that at ϕ_{us}, t_{ignit} becomes longer and longer, and approaches infinity as the values of V_{eff} at ϕ_{us} and ϕ_{other} become the same. The development of the double well potential and m_{Higgs}^2 as a function of t_{ignit} is illustrated in Fig. 1. Note that the larger the difference $|V_{\text{eff}}(\phi_{\text{other}}) - V_{\text{eff}}(\phi_{\text{us}})|$ the more the realization of, say, $I_{\text{fixed 2}}$ will in general depend on t_{ignit}. If $V_{\text{eff}}(\phi_{\text{us}}) = V_{\text{eff}}(\phi_{\text{other}})$, t_{ignit} plays no role in realizing e.g. $I_{\text{fixed 2}}$, and the value of m_{Higgs}^2 becomes independent of t_{ignit}.

6   Avoiding paradoxes arising from non-locality

In general, the presence of non-locality leads to paradoxes. While the form of the non-local action (or potential V_{nl} in this discussion) is unknown to us, we make the 4 generically representative guesses portrayed as the 4 non-locality curves in Fig. 1. In particular, non-locality curves having a negative slope as a function of t_{ignit} lead to paradoxes in the following manner. Consider the non-locality curve in Fig. 1 drawn with bold line that is redrawn in a rotated position in Fig. 2. Let us make the assumption that t_{ignit} is large and see that this leads to a contradiction. Assuming that t_{ignit} is large, it is seen from the non-locality function in Fig. 2 (call it
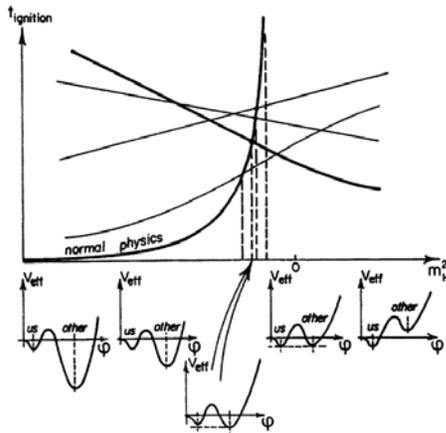
Figure 1. The development of the double well potential and m_{Higgs} as a function of t_{ignit}. Note that all the more or less randomly drawn non-locality curves intersect the "normal physics" curve near where the vacua are degenerate (i.e., the MPP solution).
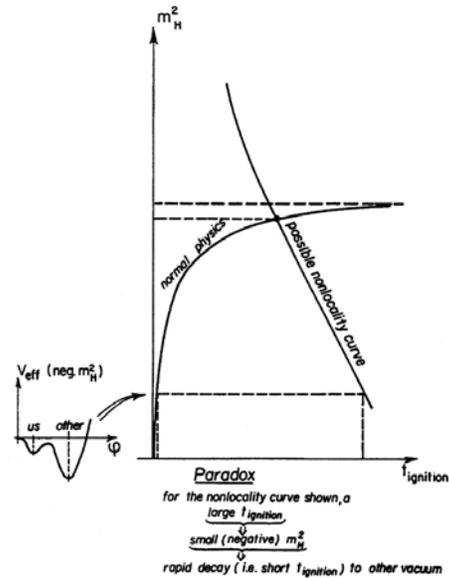

Figure 2. Many non-locality curves could lead to paradoxes similar to the "matricide" paradox. Such paradoxes are avoided if the value of m_{Higgs} is fine-tuned to the multiple point critical value. This corresponds to the intersection of the "normal physics" curve with the "possible non-locality" curve.

$m^2_{\text{Higgs nl}}(t_{\text{ignit}})$ to distinguish it from the "normal physics" $m^2_{\text{Higgs}}(t_{\text{ignit}})$) that this implies that the "normal physics" m^2_{Higgs} has a large negative value. But a large negative value of m^2_{Higgs} corresponds in "normal physics"to a (false) vacuum at ϕ_{us} that is very unstable and therefore to a very short t_{ignit} corresponding to a rapid decay to the stable vacuum at ϕ_{other}. So the paradox appears: the assumption of a *large* t_{ignit} implies a *small* t_{ignit}. This happens because in general $m^2_{\text{Higgs}}(t_{\text{ignit}}) \neq m^2_{\text{Higgs nl}}(t_{\text{ignit}})$ and is akin to the "matricide" paradox encountered for example when dealing with "time machines". It is well-known [8–10] that Nature avoids such paradoxes by choosing a very clever solution in situations where these paradoxes lure.

In the case of the paradoxes that can come about due to non-locality of the type considered here, a clever solution that avoids paradoxes is available to Nature in the form of the Multiple Point Principle (MPP). The MPP solution corresponds to the intersection of the "normal physics" curve and the "non-locality curve" in Fig. 2. because here the vacua at ϕ_{us} and ϕ_{other} are (essentially) degenerate. But at this intersection point, $m^2_{\text{Higgs}}(t_{\text{ignit}}) = m^2_{\text{Higgs nl}}(t_{\text{ignit}})$ so the paradox is avoided. So the paradox is avoided at the multiple point. But at the multiple point, an intensive parameter has its value fine-tuned for a wide range of values of the conjugate extensive quantity. Fine-tuning can therefore be understood as a consequence of Nature's way of avoiding paradoxes that can come about due to non-locality.

It should be pointed out that the paradox-free solution corresponding to the intersection of the two curves in Fig. 2 occurs for a value of m^2_{Higgs} corresponding to "our" vacuum at ϕ_{us} being very slightly unstable. The value of m^2_{Higgs} corresponding to the vacua at ϕ_{us} and ϕ_{other} being (precisely) degenerate is slightly less negative than that corresponding to the multiple point value of m^2_{Higgs} at the intersection of the curves. Note that the multiple point value of m^2_{Higgs} is very insensitive to which "guess" we use for the non-local action. Indeed all the "non-locality" curves in Fig. 1 intersect the "normal physics" curve at values of m^2_{Higgs} that are tightly nested together. The reason for this is that m^2_{Higgs} is a very slowly varying function of t_{ignit} as

$m_{\mathrm{Higgs}}^2(t_{\mathrm{ignit}})$ approaches the value corresponding to degenerate vacua. The more nearly parallel the "normal physics" and the "non-locality" curves at the point of intersection, the smaller are the (paradoxical) effects of non-locality. For a point of intersection at values of t_{ignit} sufficiently large that (the "normal physics") $m_{\mathrm{Higgs}}^2(t_{\mathrm{ignit}}) \approx m_{\mathrm{Higgs}}^2(\infty)$, the non-locality effects disappear as the curves become parallel since both curves become independent of t_{ignit}. If the curves were parallel, there would also be the possibility that these do not intersect, in which case there would be no "miraculous solution" that could avoid the paradoxes imbued in having non-locality.

If the interval $|\phi_{\mathrm{other}}^2 - \phi_{\mathrm{us}}^2|$ is large (e.g. of the order of the largest physically conceivable scale (Planck?) if tuning is to be maximally effective) and if $I_{\mathrm{fixed}\,2}$ falls not too close to the ends of this interval, then t_{ignit} will be something of the order of half the life of the universe. Actually, the approximate degeneracy of the vacua $V_{\mathrm{eff}}(\phi_{\mathrm{us}}) \approx V_{\mathrm{eff}}(\phi_{\mathrm{other}})$ may be characteristic of the temperature of the post-Big Bang universe in the present epoch and *not* characteristic of the high temperature that prevailed immediately following the Big Bang. At high temperatures, the free energy is considerably smaller than the total energy if the entropy is large enough. A phase with a large number of light particles – for example a Coulomb-like vacuum such as the "us" phase in which we live – could very plausibly be so strongly favoured at high temperatures that other phases – for example the "other" vacuum – simply disappeared at the high temperature of the universe immediately following the Big Bang.

If this were to have depleted the universe of the phase having ϕ_{other} at high temperatures, it would indeed be difficult to re-establish it in a lower temperature universe even if the vacuum at ϕ_{us} were to be only meta-stable and the vacuum at ϕ_{other} were the true vacuum at the lower temperature. Such an exchange of the true vacuum is indeed a possibility in going to lower temperatures inasmuch as the difference between the total energy and the free energy decreases in going to lower temperatures. Accordingly, this difference becomes less effective in favoring a Coulomb-like phase at the expense of a phase with heavier particles.

At this point we point out that when we say the "vacuum at ϕ_{us}" and "vacuum at ϕ_{other}" we are thinking of the approximation $\phi = \phi_{\mathrm{us}}$ and $\phi = \phi_{\mathrm{other}}$ almost everywhere, and at all times in respectively the early and late epochs of the universe in our discussion. More correctly, we should talk about vacuum densities $\langle \phi(x^0, \vec{x}) \rangle_{\mathrm{us}}$ and $\langle \phi(x^0, \vec{x}) \rangle_{\mathrm{other}}$ where

$$\langle \phi(x^0, \vec{x}) \rangle_{\mathrm{us}} \stackrel{\mathrm{def}}{=} \frac{1}{V_{\mathrm{us}}} \int_{t_{BB}}^{t_{\mathrm{ignit}}} dx^0 \int d^3\vec{x} \sqrt{g(x)} \phi(x^0, \vec{x}),$$

where V_{us} denotes the the 4-volume of the universe in the first epoch. $\langle \phi(x^0, \vec{x}) \rangle_{\mathrm{other}}$ is defined analogously. The more correct $\langle \phi(x^0, \vec{x}) \rangle_{\mathrm{us}}$ is mentioned here so as not to confuse the reader when we talk about changes in $V_{\mathrm{eff}}(\phi_{\mathrm{us}}(x^0, \vec{x}))$ as the universe cools following Big Bang.

Recall now that the value of say of $I_{\mathrm{fixed}\,2}$ can easily (i.e. as a generic possibility) assume a value that requires that the universe to be in the "phase" with $\langle \phi(x^0, \vec{x}) \rangle_{\mathrm{other}}$ during a sizeable part of its life, if the universe is to have multiple point parameters in the course of its evolution (as required for avoiding the paradoxes that accompany non-locality). How can Nature overcome the energy barrier that must be surmounted in order to bring about the decay of the slightly unstable (false) vacuum with $\langle \phi(x^0, \vec{x}) \rangle_{\mathrm{us}}$ to the vacuum with $\langle \phi(x^0, \vec{x}) \rangle_{\mathrm{other}}$? Even producing just a tiny "seed" of the "true" vacuum having $\langle \phi(x^0, \vec{x}) \rangle_{\mathrm{other}}$ would be very difficult. What miraculously clever means can Nature devise so as to avoid deviations from a multiply critical evolution of the universe? One ingenious master plan that Nature may have implemented is the creation of life with the express "purpose" of evolving some (super intelligent?) physicists that could ignite a "vacuum bomb" by first creating in some very expensive accelerator the required "seed" of the "correct" vacuum having $\langle \phi(x^0, \vec{x}) \rangle_{\mathrm{other}}$ that subsequently would engulf the universe in a (for us) cataclysmic transition to the "other" phase thereby permitting the continued evolution of a "paradox-free" universe!

7   Conclusion

We attempt to justify the assertion that fine-tuning in Nature seems to imply a fundamental form of non-local interaction. This could be manifested in a phenomenologically acceptable form as everywhere in space-time identical interactions between any pair of space-time points. This would be implemented by requiring the non-local action to be diffeomorphism invariant.

Next we put forth our multiple point principle which states that coupling parameters in the Standard Model tend to assume values that correspond to the values of action parameters lying at the junction of a maximum number of regulator induced phases (e.g., so-called "lattice artifact phases") separated from one another in action parameter space by first order transitions. The action which of course is defined on a gauge group (e.g., the non-simple SM gauge group) governs fluctuation patterns along the various subgroup combinations (K, H) with $H \triangleleft K \subseteq G$ that characterize the phases that come together at the multiple point. We then consider extensive quantities that are functions of functionals $I_{f_j}[\phi(x)]$ that are essentially Feymann path histories of the Universe for functions $f_j(\phi)$ of the fields $\phi(x)$ and derivatives of these fields. We then think of the generic situation in which these extensive quantities can happen to be fixed at values that require the universe to be realized as two or more coexisting phases. We draw on the analogy to the forced coexistence of ice and liquid water that occurs for a whole range of possible total energies because of the finite heat of melting (first order phase transition). With our multiple point principle, the intensive quantities (couplings) conjugate to extensive quantities fixed in this way become fine-tuned in a manner analogous to the fine tuning of temperature to $0°$ C when the total energy of a system of H_2O can only be realized as coexisting ice and liquid phases.

One generic way of having coexisting phases in a quantum field theory in 3+1 dimensions would be to have different phases in different epochs of the lifetime of the Universe with phase transitions occurring at various times in the course of the lifetime of the Universe. If the transitions were first order, one would have fine-tuning of (intensive) couplings conjugate to extensive quantity values that can only be realized by having coexisting (i.e., more than one) phases. But such a fine-tuning would involve non-locality: the fine-tuned values of coupling constants would depend on future phase transitions into phases that do not even exist at the time such couplings are fine-tuned.

Even non-locality of this sort (i.e., non-locally manifested as a diffeomorphism invariant non-local action) can lead to paradoxes of the "matricide paradox" type. We argue that such paradoxes are avoided when Nature chooses the multiple point principle solution to the problem of fine-tuning.

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