Proceedings of Institute of Mathematics of NAS of Ukraine 2004, Vol. 50, Part 3, 1051–1052

On the Gauge Groups of Particle Physics and Their Quantum Deformations[1]

Jules BECKERS

Institute of Physics, University of Liege, Liege, Belgium
E-mail: *Jules.Beckers@ulg.ac.be*

All the forces of Nature are actually understood as mediated by an exchange of gauge fields of the symmetry group $SU(3) \otimes SU(2) \otimes U(1)$. Such a gauge principle then describes the very important field called "particle (or high energy) physics" and its famous "*Standard Model*" relating strong, electromagnetic and weak (or strong and electroweak) interactions. Knowing that the *quantum* deformations of such unitary local symmetry groups as well as their irreducible representations have been studied in a relatively recent past, we propose to visit a corresponding "*q-Standard Model*". Such a step is seen as the first part of our general programme and will be useful in a second one coming back on "old current algebras and associated sum rules" based on *chiral* symmetry groups such as $SU(2) \otimes SU(2)$ and $SU(3) \otimes SU(3)$. Here we plan to introduce the main useful ingredients for these purposes but by taking the q-deformed version of $SU(2)$ as the typical example.

The series of unitary $U(n, \mathbb{C})$- or more often unimodular and unitary $SU(n, \mathbb{C})$-Lie groups have already played very important roles in particle physics by characterizing weak, electromagnetic and (or) strong interactions at different occasions. If we limit ourselves to recalling the famous gauge principle [1, 2] saying that all the forces of Nature are actually understood as mediated by an exchange of gauge fields, we immediately think of the following direct product of Lie groups $SU(3) \otimes SU(2) \otimes U(1)$, where $SU(3)$ refers to the colour group of QCD (Quantum Chromodynamics) for strong interactions while $SU(2) \otimes U(1)$ is necessary for taking account of electroweak ones. More recently since the pioneering works of Jimbo [3] and Drinfeld [4], quantum groups and their representations have been extensively studied so that their impact inside particle physics developments [5] is a natural inquiry in order to exploit new possibilities according to recent experimental results.

In particular, the quantum deformation of $SU(2)$ has to be visited in connection with the electroweak interactions as already suggested by Finkelstein [6] for example but also inside chiral algebras (and their associated sum rules) such as $SU(2) \times SU(2)$ developed elsewhere [7].

This lecture is mainly devoted to the introduction of some necessary and useful ingredients dealing with the standard model on the one side and with the representations of the quantum deformation of $SU(2)$ on the other one in order to enter our general programme inside the chiral algebras more particularly.

We insist on Q.E.D. (Quantun Electrodynamics) seen as an Abelian gauge theory based on the gauge group $U(1)$. Then we go to the non-Abelian context through the example of Yang–Mills developments based on the gauge group $SU(2)$. One of the main points necessary for the understanding of the standard model is the introduction and the construction of current densities inside the Lagrangian of the model describing electroweak interactions: we thus come back on the Puppi triangle for simplicity but also on the Gell-Mann tetrahedron in weak interaction. Finally some information on quantum deformations (their generalities and representations) may be given by taking the simplest context of q-$SU(2)$ as the example of particular interest.

Such ingredients being essentially proposed for information to non-specialists in the already mentioned fields, we have decided to give here some specific references where the useful tools can be found and understood while the contents of this report are limited.

[1] It is short review of plenary talk given by Prof. J. Beckers at the Conference.

We want to insist on the very compact and precise overview of the standard model reported by Kazakov [2]. Its necessary elements can be found in standard textbooks in Q.E.D., in Q.C.D. and in weak interactions like those described by Weinberg [1] and by Marshak et al. [8], for example, that I strongly recommend. There the introduction of covariant derivatives and the way to understand how the gauge principle is a dynamical one are very simple, as well as the extension of Abelian to non-Abelian developments according to those given by Yang–Mills [9], for example. In the same context the very well-known beta-decay can help us to understand the construction of current densities in the hadronic and leptonic worlds by including not only Dirac matrices but also isospin ones inside these densities entering into ad-hoc Lagrangians for the V-A theory of weak interactions.

Trough the simplest non-Abelian Lie algebra corresponding to the group $SU(2,\mathbb{C})$ as an example, we have also shown how to get very easily its quantum group by replacing the Jacobi identity by the famous Yang–Baxter equation in order to get the typical entries of the corresponding quantum deformation of $SU(2,\mathbb{C})$ and, more importantly its q-Wigner functions entering in the corresponding irreducible representations.

Then let us take the 2-dimensional Borel factorization in order to show how to obtain, following Finkelstein [6], the corresponding dual algebras of special interest here. After Finkelstein we get the q-dependent commutation relations varying with the $(2j+1)$-dimensional representation. We see that the fundamental 2-dimensional representation is not altered but that the 3-dimensional adjoint one is: this is the important property by going back to the Standard Model where we have to recall that the (spin 1/2) fermion sector corresponding to the matter fields (leptons and quarks) deals with the fundamental representation(s) of the gauge group(s) while the (spin 1) gauge sector corresponding to the gauge bosons (spin 1-vector particles) deals with their adjoint representation(s). Such a last point has an important impact in the corresponding gauge theory and could be included by q-modifying the corresponding "structure constants" typical of such representations in accordance with the existing q-modified commutation relations in that "adjoint" context in particular!

This motivates the q-electroweak theory developed by Finkelstein [6]: the corresponding Lagrangian density consequently contains quadrilinear terms (Section 5) showing such a q-dependence which should be related to "experimental results" appearing, maybe, in particular in an isotopic context.

Acknowledgements

It is our pleasure to thank the Belgian "F.N.R.S." for a financial support in order to participate to this Kyiv Conference.

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