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Extended Poincaré Parasuperalgebra with Central Charges and Wess-Zumino-Weinberg Model

A.G.Nikitin* and A.V.Galkin†

Institute of Mathematics, National Academy of Sciences of Ukraine,
3 Tereshenkivska str., Kiev-4, Ukraine, 01601

We describe irreducible representations of the extended Poincaré parasuperalgebra (PPSA) which includes an arbitrary number N of parasupercharges, n ($n \leq \{\frac{N}{2}\}$) central charges and internal symmetry group. We also propose a parasupersymmetric generalization of the Wess-Zumino-Weinberg model for arbitrary p and N.

1. Introduction

In the present paper we generalize the results obtained in works [1-5] in which the concept of the Poincaré parasuperalgebra (PPSA) was proposed and the group-theoretical foundations of parasupersymmetric quantum field theory (PSSQFT) were formulated. Here we consider the irreducible reperesentations of PPSA with an arbitrary N of parasupercharges, nontrivial central charges and internal symmetry algebra.

The PPSA is a direct generalization of the Poincaré superalgebra (PSA) with the central charges [6]. But this generalization is by no means trivial. A new feature of the extended PPSA is the existence of hermitian IRs which correspond to values of central charges larger then doubled mass.

We present also a parasupersymmetric generalization of the Wess-Zumino -Weinberg model for arbitrary number N of parasupercharges and arbitrary value of paraquantization parameter p.

2. Extended Poincaré Parasuperalgebra

The Poincaré prasuperalgebra [1-5] includes ten even elements (generators of the Poincaré group) P_μ, $J_{\mu\nu}$, $\mu,\nu = 0,1,2,3$, satisfying the commutation relations

$$\begin{aligned}
&[P_\mu, P_\nu] = 0,\\
&[P_\mu, J_{\nu\sigma}] = i(g_{\mu\nu}P_s - g_{\mu\sigma}P_\nu),\\
&[J_{\mu\nu}, J_{\rho\sigma}] = i(g_{\mu\sigma}J_{\nu\rho} + g_{\nu\rho}J_{\mu\sigma}\\
&\qquad - g_{\mu\rho}J_{\nu\sigma} - g_{\nu\sigma}J_{\mu\rho})
\end{aligned} \tag{2.1}$$

and N parasupercharges Q^j_A, $(Q^j_A)^\dagger$ ($A = 1,2$; $j = 1,2,\ldots,N$), which satisfy the following double commutation relations

$$\begin{aligned}
&[Q^i_A, [Q^j_B, Q^k_C]]\\
&= 2\varepsilon_{AB}Z^{ij}Q^k_C - 2\varepsilon_{AC}Z^{ik}Q^j_B,\\
&[(Q^i_A)^\dagger, [(Q^j_B)^\dagger, (Q^k_C)^\dagger]]\\
&= 2\varepsilon_{AB}Z^{*ij}(Q^k_C)^\dagger - 2\varepsilon_{AC}Z^{*ik}(Q^j_B)^\dagger,\\
&[Q^i_A, [Q^j_B, (Q^k_C)^\dagger]]\\
&= 2\varepsilon_{AB}Z^{ij}(Q^k_C)^\dagger - 4Q^j_B(\sigma_\mu)_{AC}P^\mu\delta^{ik},\\
&[(Q^i_A)^\dagger, [Q^j_B, (Q^k_C)^\dagger]]\\
&= 4(Q^k_C)^\dagger(\sigma_\mu)_{AB}P^\mu\delta^{ij} - 2\varepsilon_{AC}Z^{*ik}Q^j_B
\end{aligned} \tag{2.2}$$

where σ_ν are the Pauli matrices, ε_{AB} is the universal spinor $\varepsilon_{11} = \varepsilon_{22} = 0$, $\varepsilon_{12} = -\varepsilon_{21} = 1$; $(.)_{AC}$ relate to matrix elements. Relations (2.1), (2.2) include operators Z^{ij} which we call central charges.

Like the case of Poincaré superalgebra the central charges are supposed to satisfy the relations $(Z^{ij})^* = Z^{ij}$ and $Z^{ij} = -Z^{ji}$ and to commute with generators of the PPSA.

The parasupercharges are supposed to be Weyl spinors and so they should satisfy the following relations:

$$\begin{aligned}
&[J_{\mu\nu}, Q^j_A] = -\tfrac{1}{2i}(\sigma_{\mu\nu})_{AB}Q^j_B,\\
&[P_\mu, Q^j_A] = 0,\\
&[J_{\mu\nu}, (Q^j_A)^\dagger] = -\tfrac{1}{2i}(\sigma^*_{\mu\nu})_{AB}(Q^j_B)^\dagger,\\
&[P_\mu, (Q^j_A)^\dagger] = 0
\end{aligned} \tag{2.3}$$

where $\sigma_{\mu\nu} = \frac{i}{2}[\sigma_\mu,\ \sigma_\nu]$.

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The extended PPSA is a direct generalization of the Poincaré superalgebra. Indeed, it is possible to show that relations (2.1)-(2.3) are valued for basis elements of the PSA but are more weak than usual relations for PSA [7]

Like the Poincaré superalgebra the PPSA can be extended by adding the generators Σ_l of the internal symmetry group which satisfy the following relations:

$$\begin{aligned}
&[Q^i_A, \Sigma_l] = T_l^{ij} Q^j_A,\\
&[\Sigma_l, (Q^i_A)^\dagger] = T_l^{*ij}(Q^j_A)^\dagger\\
&[\Sigma_l, \Sigma_m] = f^k_{lm}\Sigma_k,
\end{aligned} \tag{2.4}$$

where f^k_{lm} are structure constants of the internal symmetry group. The constants T^I_{lJ} are specified in the following.

3. Classification of IRs and Explicit Form of Basis Elements and Internal Symmetry group

The Casimir operators for the extended PPSA are [1]

$$\begin{aligned}
&C_1 = P_\mu P^\mu,\\
&C_2 = P_\mu P^\mu B_\nu B^\nu - (B_\mu P^\mu)^2,
\end{aligned} \tag{3.1}$$

where

$$\begin{aligned}
&B_\mu = W_\mu + X_\mu,\\
&W_\mu = \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma}J^{\nu\rho}P^\sigma, X_\mu = (\sigma_\mu)_{AB}(Q^i_A)^\dagger Q^i_B.
\end{aligned}$$

Like the case of the ordinary Poincaré group [8], IRs of the PPSA are qualitatively different for the following cases

$$\begin{aligned}
\text{I.}\quad & P_\mu P^\mu = M^2 > 0,\\
\text{II.}\quad & P_\mu P^\mu = 0,\\
\text{III.}\quad & P_\mu P^\mu < 0.
\end{aligned}$$

For the cases I and II there exists the additional Casimir operator $C_3 = P_0/|P_0|$ whose eigenvalues are $\varepsilon = \pm 1$. Here we consider only such representations which correspond to $C_1 > 0$ and $C_3 > 0$. This class of representations will be denoted as I^+.

Considering these relations in the rest frame $P = (M, 0, 0, 0)$ we conclude that the related three-vector

$$j_k = -\frac{B_k}{M} = S_k - \frac{X_k}{M}, \quad k = 1, 2, 3 \tag{3.2}$$

commutes with Q^j_A, $(Q^j_A)^\dagger$ and satisfies the commutation relation which characterize algebra $so(3)$:

$$\begin{aligned}
&[j_a, \hat{Q}^j_A] = [j_a, \hat{\bar{Q}}^j_A] = 0,\\
&[j_a, j_b] = i\varepsilon_{abc} j_c.
\end{aligned} \tag{3.3}$$

Let the central charges are nontrivial, then using the unitary transformation

$$\begin{aligned}
&Q^i_A \longrightarrow \tilde{Q}^j_A = U^{ij} Q^j_A,\\
&Z^{ij} \longrightarrow \tilde{Z}^{ij} = U^{ik}U^{jl}Z^{kl},\\
&J_{\mu\nu} \longrightarrow J_{\mu\nu}, \quad P_\mu \longrightarrow P_\mu,
\end{aligned} \tag{3.4}$$

we can reduce the antisymmetric matrix Z^{ij} to the quasidiagonal representation such that all nonzero elements have the following form

$$\tilde{Z}^{2m-1,2m} = -\tilde{Z}^{2m,2m-1} = Z_m, \tag{3.5}$$

where Z_m, $m = 1, 2, ..., \{\frac{N}{2}\}$, are real and nonnegative values.

Denoting $(\hat{Q}^i_A)^\dagger = \hat{\bar{Q}}^i_A$ and choosing a new basis

$$\begin{aligned}
\tilde{Q}^{2m-1}_1 &= \frac{1}{\sqrt{2}}(\hat{Q}^{2m-1}_1 - \hat{\bar{Q}}^{2m}_2),\\
\tilde{Q}^{2m-1}_2 &= \frac{1}{\sqrt{2}}(\hat{Q}^{2m}_1 + \hat{\bar{Q}}^{2m-1}_2),\\
\tilde{Q}^{2m}_1 &= \frac{1}{\sqrt{2}}(\hat{Q}^{2m-1}_1 + \hat{\bar{Q}}^{2m}_2),\\
\tilde{Q}^{2m}_2 &= \frac{1}{\sqrt{2}}(\hat{Q}^{2m}_1 - \hat{\bar{Q}}^{2m-1}_2)
\end{aligned} \tag{3.6}$$

we reduce relations (2.2) in the rest frame $P = (M, 0, 0, 0)$ to the form

$$\begin{aligned}
&[\hat{Q}^{2k-1}_A, [\hat{\bar{Q}}^{2m-1}_B, \hat{Q}^j_C]]\\
&= 2\delta_{AB}\delta_{km}(2M - Z_m)\hat{Q}^j_C,\\
&[\hat{\bar{Q}}^{2k-1}_A, [\hat{Q}^{2m-1}_B, \hat{\bar{Q}}^j_C]]\\
&= 2\delta_{AB}\delta_{km}(2M - Z_m)\hat{\bar{Q}}^j_C,
\end{aligned} \tag{3.7a}$$

$$\begin{aligned}
&[\hat{Q}^{2k}_A, [\hat{\bar{Q}}^{2m}_B, \hat{Q}^j_C]]\\
&= 2\delta_{AB}\delta_{km}(2M + Z_m)\hat{Q}^j_C,\\
&[\hat{\bar{Q}}^{2k}_A, [\hat{Q}^{2m}_B, \hat{\bar{Q}}^j_C]]\\
&= 2\delta_{AB}\delta_{km}(2M + Z_m)\hat{\bar{Q}}^j_C
\end{aligned} \tag{3.7b}$$

the remaining double commutators of the parasupercharges are equal to zero.

Thus description of IRs of the extended PPSA, belonging to Class I$^+$, reduces to description of representations of the little Wigner parasuperalgebra (LWPSA) defined by relations (3.3), (3.7). In accordance with (3.3) the LWPSA is a direct sum of algebra $so(3)$ (realized by j_1, j_2 and j_3) and the algebra formed by parasupercharges.

It is well known that hermitian IRs of the extended Poincaré superalgebra can be defined only in the case when values of supercharges do not exceed $2M$ [6]. We will see that in the case of PPSA such IRs exist for any real values of Z_m.

We specify the following cases

1. The central charges are trivial i.e., $Z_m = 0$, $m = 1, 2, ..., \{\frac{N}{2}\}$.

2. The central charges are nontrivial and are smaller then $2M$.

3. The central charges are equal to $2M$.

4. The central charges are nontrivial and their values exceed $2M$.

5. The central charges are of mixed type, i.e.,

$$\begin{gathered} 0 < Z_i < 2M, \quad i = 1, 2, ..., m_1; \\ Z_{m_1+j} > 2M, \quad j = 1, 2, ..., m_2; \\ Z_{m_1+m_2+k} = 2M, \quad k = 1, 2, ..., m_3; \\ Z_{m_1+m_2+m_3+l} = 0, \\ l = 1, 2, ..., \{\tfrac{N}{2}\} - m_1 - m_2 - m_3. \end{gathered} \tag{3.8}$$

Consider all these possibilities consequently. Of course Cases 1-4 are particular versions of Case 5.

Representations corresponding to the trivial central charges are described in papers [3-5]. The related WLPSA can be enclosed into the direct sum of the orthogonal algebras

$$WLPSA \subset so(4N + 1) \oplus so(3) \tag{3.9}$$

and so IRs of the PPSA are induced by IRs of $so(4N + 1)$ and $so(3)$. They are labeled by the sets of numbers $(M, j, n_1, n_2, ..., n_{2N})$ where $n_1 \geq n_2 \geq n_3 \geq ... \geq n_{2N} \geq 0$ are both integers or half integers, j is an integer or half integer. The explicit form of the corresponding basis elements $P_\mu, J_{\mu\nu}, Q^i_A, \bar{Q}^j_A$ in the momentum representation up to unitary equivalence can be chosen as [3-5]

$$\begin{gathered} P_0 = \varepsilon E, \qquad P_a = p_a, \\ J_{ab} = x_a p_b - x_b p_a + \varepsilon_{abc} S_c, \\ J_{0a} = x_0 p_a - \frac{i\varepsilon}{2}[\frac{\partial}{\partial p_a}, E]_+ - \frac{\varepsilon_{abc} p_b S_c}{E + M}. \end{gathered} \tag{3.10a}$$

$$\begin{gathered} Q^i_1 = \frac{[\tilde{Q}^j_1(E + M + p_3) + \tilde{Q}^j_2(p_1 - ip_2)]}{\sqrt{2M(E + M)}}, \\ Q^i_2 = \frac{[\tilde{Q}^j_1(p_1 + ip_2) + \tilde{Q}^j_2(E + M - p_3)]}{\sqrt{2M(E + M)}}, \\ \bar{Q}^i_A = (Q^i_A)^\dagger, \quad i = 1, 2, ..., N \end{gathered} \tag{3.10b}$$

where $E = \sqrt{M^2 + p^2}$, $x_a = i\frac{\partial}{\partial p_a}$; S_a and $\tilde{Q}^j_A$ $(a = 1, 2, 3; j = 1, 2, ..., N; A = 1, 2)$ are matrices given by the following relations

$$\begin{gathered} \tilde{Q}^j_1 = \sqrt{2M}(S_{\Lambda,\, 4j-3+p} - iS_{\Lambda,\, 4j-2+p}), \\ \tilde{Q}^j_2 = \sqrt{2M}(S_{\Lambda,\, 4j-1+p} - iS_{\Lambda,\, 4j+p}), \end{gathered} \tag{3.11}$$

$$S_a = S^{(1)}_a \oplus j_a,$$

$$\begin{aligned} S^{(1)}_1 &= \frac{1}{2}\sum_{i=1}^{N}(S_{4j,\, 4j-3} + S_{4j-2p,\, 4j-1}), \\ S^{(1)}_2 &= \frac{1}{2}\sum_{i=1}^{N}(S_{4j,\, 4j-2} + S_{4j-1,\, 4j-3}), \\ S^{(1)}_3 &= \frac{1}{2}\sum_{i=1}^{N}(S_{4j,\, 4j-1} + S_{4j-3,\, 4j-2}). \end{aligned} \tag{3.12}$$

Here $\Lambda = 4N + 1$, S_{mn} are generators of algebra $so(4N + 1)$.

We see IRs of the PPSA belonging to Class I$^+$ can be described in rather straightforward way. For other classes of representations refer to [3-5].

For the case 2-4 where the central charges are nontrivial the WLPSA reduces to the following algebras [12]

$Z_m < 2M$ *and* $Z_m > 2M$

$N = 2k$

$WLPSA \subset so(2N, 2N + 1) \oplus so(3)$

$N = 2k + 1$

$WLPSA \subset so(2N - 4, 2N + 5) \oplus so(3)$

$Z_m = 2M$,

$WLPSA \subset so(2N + 1) \oplus so(3)$

In the general case 5 the WLPSA is $so(2m_2, 4N-2m_3-2m_2+1)\oplus so(3)$.

Thus in contrast with the PSA there exist hermitian IRs of the PPSA corresponding to larger then $2M$ central charges.

New let us demonstrate that the space of IR of the extended PPSA is a carrier space for the internal symmetry group, and construct explicitly the related group generators.

If central charges are trivial then commutation relations (2.1)- (2.3) are transparently invariant w.r.t. unitary transformations

$$Q_A^j \longrightarrow U^{jk}Q_A^k, \quad J_{\mu\nu} \longrightarrow J_{\mu\nu}, \quad P_\mu \longrightarrow P_\mu, \\ U^{jk}(U^{ik})^\dagger = \delta^{ij} \tag{3.15}$$

and so the corresponding PPSA admits the internal symmetry group $U(N)$.

If central charges are nontrivial then the internal symmetry group is less extended. Indeed, consider the first of relations (2.2) for $A=C=1$, $B=2$:

$$[Q_1^i,[Q_2^j,Q_1^k]] = 2Z^{ij}Q_1^k. \tag{3.16}$$

Calculating commutators of the l.h.s and r.h.s. of (3.16) with Σ_l and using (2.4) we come to the following condition

$$T_l^{ij}Z^{jk} = T_l^{kj}Z^{ji} = (T_l^{ij}Z^{jk})^\dagger. \tag{3.17}$$

In other words, a product of a generator of the internal symmetry group with the matrix of central charges should be a symmetric and hermitian matrix.

In the case of N even and all $Z_m \neq 0$ relation (3.17) specifies algebra $sp(\{\frac{N}{2}\})$. For N odd or for Z_m of combined type (3.8) the matrix Z^{kl} is equivalent to the direct sum of the invertible antisymmetric matrix and the zero matrix and the related conditions (3.15), (3.17) specify the direct sum of algebras $sp(m_1+m_2+m_3)\oplus u(N-2(m_1+m_2+m_3))$.

Thus the structure of internal symmetries for the PPSA is clear and is analogous to the case of PSA [6].

4. Wess-Zumino-Weinberg Model for arbitrary p,N and Z

Here we present a formal construction of nonlinear models which generalize both the WZ and Weinberg [10] approaches to the case of parasuperfield with arbitrary p, N and Z.

To construct this model we notice that extended PPSA admits a special relization which called covariant [3,4]. In this realization generators of the Poincaré group and parasupercharges have the following form

$$P_\mu = p_\mu, \qquad J_{\mu\nu} = x_\mu p_\nu - x_\nu p_\mu + S_{\mu\nu}, \tag{4.1}$$

$$\begin{aligned} Q_1^j &= S_{4N+1\ 4j-3} - iS_{4N+1\ 4j-2}, \\ Q_2^j &= -S_{4N+1\ 4j-1} + iS_{4N+1\ 4j}, \\ \bar{Q}_1^j &= (S_{4N+1\ 4j-3} - iS_{4N+1\ 4j-2})(p_3-p_0) \\ &\quad +(S_{4N+1\ 4j-1} + iS_{4N+1\ 4j})(p_1 - ip_2), \\ \bar{Q}_2^j &= -(S_{4N+1\ 4j-1} + iS_{4N+1\ 4j})(p_3+p_0) \\ &\quad +(S_{4N+1\ 4j-3} - iS_{4N+1\ 4j-2})(p_1 + ip_2). \end{aligned} \tag{4.2}$$

Here $S_{\mu\nu}$ are numerical matrices which commute with the orbital part of $J_{\mu\nu}$:

$$S_{ab} = \varepsilon_{abc}S_c^{(1)}, \\ S_{0a} = iS_a^{(1)}, \quad a,b,c = 1,2,3, \tag{4.3}$$

and $S_a^{(1)}$ are matrices defined in (3.12).

It was shown in [5] that realization (4.1)-(4.3) is equivalent to (3.10). Matrices $S_{\mu\nu}$ and parasupercharges Q_A^j, $\bar{Q}_A^j$ can be represented in terms of paragrassmanian variables [10].

$$\begin{aligned} S_{\mu\nu} &= \frac{1}{2}\sum_{j=1}^{N}\left((\sigma_{\mu\nu})_{AB}\left[\theta_A^j, \frac{\partial}{\partial\theta_B^j}\right]\right. \\ &\quad \left. +(\sigma^\dagger{}_{\mu\nu})_{AB}\left[(\theta_A^j)^\dagger, \frac{\partial}{\partial(\theta_B^j)^\dagger}\right]\right), \\ Q_A^i &= \sqrt{m}\frac{\partial}{\partial\theta_A^i} + \frac{1}{\sqrt{m}}(\sigma_\mu)_{AB}(\theta_B^i)^\dagger P^\mu \\ &\quad +\frac{1}{\sqrt{m}}\widehat{Z}^{ij}\varepsilon_{AB}\theta_B^j, \\ (Q_A^i)^\dagger &= \sqrt{m}\frac{\partial}{\partial(\theta_A^i)^\dagger} + \frac{1}{\sqrt{m}}\theta_B^i(\sigma_\mu)_{AB}P^\mu \\ &\quad +\frac{1}{\sqrt{m}}\widehat{Z}^{ij}\varepsilon_{AB}(\theta_B^j)^\dagger, \end{aligned} \tag{4.4}$$

where $\sigma_{0a}=\sigma_a$, $\sigma_{ab}=\frac{1}{2}\left(\sigma_a\sigma_b-\sigma_b\sigma_a\right)$, $a,b\neq 0$ and $\widehat{Z}^{ij}$ is the matrix whose elements are given in (3.5). The corresponding generators of the Poincaré group are still given by relations (4.1), (4.2). We define the corresponding representation space as a parasuperfield [11] $\Phi(x,\theta^i,(\theta^i)^\dagger)$ depending on spatial variables x_μ and paragrassmanian variables θ^i, $(\theta^i)^\dagger$. This space is reducible w.r.t. PPSA in as much as it is possible to impose on $\Phi(x,\theta^i,(\theta^i)^\dagger)$ one of the following invariant conditions

$$\overline{D^i}_A\Phi(x,\theta,(\theta)^\dagger)=0 \qquad (4.5)$$

or

$$D^i_A\Phi(x,\theta,(\theta)^\dagger)=0 \qquad (4.6)$$

where

$$\begin{aligned} D^i{}_A &= \frac{\partial}{\partial\theta^i{}_A}-(\sigma_\mu)_{AB}(\theta^i{}_B)^\dagger P^\mu, \\ \overline{D}^i{}_A &= \frac{\partial}{\partial(\theta^i{}_A)^\dagger}-\theta^i{}_B(\sigma_\mu)_{AB}P^\mu \end{aligned} \qquad (4.7)$$

are covariant derivatives.

Imposing the invariant conditions

$$\left(\varepsilon^{AB}D^j_AD^j_B\right)\Phi=0$$

(no sum over j) on the related parasuperfield $\Phi=\Phi\left(\theta^i_B,\left(\theta^j_A\right)^\dagger,x\right)$ we conclude the field Φ_+ which does not depend on θ^j_A. The corresponding invariant equation again can be written in the form

$$\begin{aligned} &\left([\overline{D}_1',\overline{D}_2']\right)^p\exp(-2G)\Phi_+^*(x,\theta) \\ &\quad=m\Phi_++g\Phi_+^2(x,\theta) \end{aligned} \qquad (4.8)$$

where

$$\begin{aligned} G=&\frac{1}{2m}\sum_{j=1}^{N}(\sigma_\mu)_{AB}[\left(\theta^j_A\right)^\dagger,\theta^j_B]P^\mu \\ &-\frac{1}{2m}\sum_{i,j=1}^{N}\widehat{Z}^{ij}\varepsilon_{AB}\left[\theta^i_A,\frac{\partial}{\partial(\theta^j_B)^\dagger}\right], \end{aligned}$$

m is mass and g is interaction constant.

Lagrangian which correspond to equation (4.8) has the following form

$$\begin{aligned} \mathcal{L}=&(\Phi_+^*\exp(-2G)\Phi_+)_{\theta_1^p\theta_2^p(\theta_1^\dagger)^p(\theta_2^\dagger)^p}+ \\ &\left(\tfrac{m}{2}\Phi_+^2+\tfrac{g}{3}\Phi_+^3\right)_{\theta_1^p\theta_2^p}+\text{(h.c.)} \end{aligned}$$

Equation (4.8) can be treated as a parasupersymmetric analogue of the Weinberg equation [9] for particle with arbitrary spin.

5. Discussion

We present a description of IRs of the extended PPSA which includes ten generators of the Poincaré group, an arbitrary number N of parasupercharges, n central charges ($n\leq\{\frac{N}{2}\}$), and also internal symmetry algebra which is $sp(n)\oplus u(N-2n)$. Such rather complicated algebraic structure admits an explicit description in terms of generators of the little Wigner parasuperalgebra which is equivalent to the direct sum of algebras $so(3)$ and $so(p,\ q)$ were $p=4N-2m_2-2m_3+1$, $q=2m_2$; m_1, m_2 and m_3 are numbers of central charges defined in (3.8).

In this way we complete investigations of IRs of the PPSA started in [3-5]. In particular, we obtain a parasupersymmetric analogue of known IRs of the PSA [6] which appear in our analysis as particular cases. Thus we present a new view point on PSA which is only the simplest link in the infinite series of the Poincaré parasuperalgebras.

A specific feature of our approach is that the basis elements of related internal symmetry algebra are given explicitly both in terms of matrices belonging to (pseudo)orthogonal algebras and in terms of paragrassmanian variables. In particular, such formulation can be useful for PSA and supersymmetric quantum field theory.

In addition, the extended PPSA admits such IRs which do not have analogues in the case of (extended) PSA. They are the representations which correspond to central charges larger then doubled mass and representations corresponding to negative eigenvalues of the Casimir operators $C_3=\frac{P_0}{|P_0|}$ or $C_2=P^\mu P_\mu$, described in [3-5].

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