The 5th International Workshop in

Group Analysis of Differential Equations and Integrable Systems

Abstracts

Protaras, Cyprus

June 6–10, 2010

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Workshop Programme

Sunday, June 6

Arrival day
20.00–21.00 *Registration*
21.30 *Welcome reception*

Monday, June 7

8.30–9.00 *Registration*

Chairperson A.G. Nikitin

9.00–9.40 **C. ROGERS** The pulsrodon in 2+1-dimensional magnetogasdynamics and Hamiltonian Ermakov reduction
9.40–10.20 **S. ANCO** Group invariant soliton equations and bi-Hamiltonian geometric curve flows in Klein geometry
10.20–11.00 **M.C. NUCCI** The unreasonable effectiveness of Lie symmetries
11.00–11.30 *Coffee break*

Chairperson M.C. Nucci

11.30–12.00 **G. CICOGNA** Lambda-symmetries of Dynamical Systems, Hamiltonian and Lagrangian equations
12.00–12.30 **S. GOLOVIN** Natural curvilinear coordinates in MHD. Flows with constant total pressure
12.30–13.00 **V. LAHNO** On realizations of Lie algebras of Poincare group and new Poincare-invariant equations
13.00–13.30 **M. ABD-EL-MALEK** Solution of Burgers' equation with time-dependent kinematic viscosity via Lie-group analysis
13.30–17.00 *Lunch break*

Chairperson J. Patera

17.00–17.20 **J. HRIVNAK** Two types of discretization of tori of compact simple Lie groups
17.20–17.40 **M. NESTERENKO** Orthogonal polynomials of compact simple Lie groups
17.40–18.00 **P. NOVOTNY** Generalized derivations, twisted cocycles and invariant functions of Lie algebras
18.00–18.20 **S. POSTA** Comparing three dimensional alternating, symmetric and antisymmetric interpolations
18.20–18.40 **L. MOTLOCHOVA** Orthogonal polynomials of symmetric and antisymmetric generalizations of trigonometric functions
18.40–19.00 **C. ACCATRINEI** A difference equation appearing in noncommutative field theory

Tuesday, June 8

Chairperson P. Damianou

9.00–9.40 **T. RATIU** Variational principles in control and template matching
9.40–10.20 **Y. SURIS** On the integrability of Hirota-Kimura discretizations
10.20–11.00 **G. CHADZITASKOS** Three boson interaction Hamiltonian solvable via Krawtchouk polynomials

11.00–11.30 *Coffee break*

Chairperson Y. Suris

11.30–12.00 **C. QU** Potential symmetries to nonlinear diffusion equations

12.00–12.30 **I. YEHORCHENKO** Reduction of multidimensional wave equations to two-dimensional equations: investigation of possible reduced equations

12.30–13.00 **L. HLAVATY** Poisson Lie symmetry and solutions of sigma models

13.00–13.30 **L. SNOBL** Symmetries and invariant solutions of PDEs on superspace

13.30–17.00 *Lunch break*

Chairperson S. Anco

17.00–17.20 **R. TRACINÀ** Fundamental solution in classical elasticity via group analysis

17.20–17.40 **S. SPICHAK** Hermitian quasi-exactly and exactly solvable matrix Schrodinger operators

17.40–18.00 **I. STEPANOVA** Equations of vibrational convection in binary mixture: symmetry properties and exact solutions

18.00–18.20 **N. IVANOVA** Group analysis of variable coefficient diffusion–convection equations

18.20–18.40 **A. BIHLO** Symmetry preserving parametrization schemes

18.40–19.00 **C. SARDON** Symmetries and reductions for the Camassa Holm hierarchy in 2+1 dim multisoliton solutions

Wednesday, June 9

Chairperson T. Ratiu

9.00–9.40 **A. NIKITIN** Group analysis and exact solutions for equations of axion electrodynamics
9.40–10.20 **R. POPOVYCH** Conservation laws and normal forms of evolution equations
10.20–11.00 **P. LEACH** Lie symmetries and certain equations of financial mathematics
11.00–11.30 *Coffee break*

Chairperson P. Leach

11.30–12.00	**G. BURDE** Conformal invariance and partially nonclassical method
12.00–12.30	**A. KISELEV** Variational Lie algebroids
12.30–13.00	**J. POHJANPELTO** Moving frames, symmetries, and reduction of exterior differential systems
13.00–13.30	**V. VLADIMIROV** Localized travelling wave solutions to the time-delayed convection-reaction-diffusion
13.30–14.30	*Lunch break*
14.30	*Excursion to Nicosia*

Thursday, June 10

Departure day

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Solution of Burgers' equation with time-dependent kinematic viscosity via Lie-group analysis

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We apply Lie-group method for determining symmetry reductions for the Burgers' equation with time-dependent kinematic viscosity. Lie-group method starts out with a general infinitesimal group of transformations under which the given partial differential equation is invariant. The determining equations are a set of linear differential equations, the solution of which gives the transformation function or the infinitesimals of the dependent and independent variables. After the group has been determined, a solution to the given partial differential equation may be found from the invariant surface condition such that its solution leads to similarity variables that reduce the number of independent variables of the partial differential equation. From which, the symmetries of the equation are determined. Using these symmetries, the resulting differential equation is solved numerically using shooting method coupled with Runge–Kutta scheme and the results are plotted.

A difference equation appearing in noncommutative field theory

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In field theory defined over a noncommutative space, bilocality and discreteness of space appear naturally. The equation for wave propagation becomes a difference equation. We discuss the solutions of this equation; their nature depends on the symmetry of the problem. No infinities appear anymore at the location of the sources.

Group invariant soliton equations and bi-Hamiltonian geometric curve flows in Klein geometry

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The sine-Gordon (SG) equation, modified Korteweg-de Vries (mKdV) equation, and nonlinear Schrodinger (NLS) equation each have a remarkable geometric origin connected with the classical frame structure equations for curve flows in S^2, R^2, and R^3, respectively. In this talk I will describe a broad generalization of these results to the setting of curve flows in Klein geometry, which gives a geometrical derivation of group-invariant (multi-component) generalizations of mKdV, NLS, and SG soliton equations along with their bi-Hamiltonian structure, symmetries, and conservation laws. The derivation uses a moving frame formulation of non-stretching curve flows and provides an explicit bi-Hamiltonian formulation for the underlying equations of motion of the curves. I will present several examples based on the geometries $S^n = SO(n+1)/SO(n)$ and $SU(n)/SO(n)$ (which are curved generalizations of Euclidean space); $HP^n = Sp(n+1)/Sp(1) \times Sp(n)$ and $SU(2n)/Sp(n)$ (which are curved quaternionic spaces).

Symmetry preserving parametrization schemes

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Although the grid resolution in modern weather and climate models is continuously increased, there is still the necessity to accurately describe the behavior of physical processes taking place below the smallest scales explicitly resolved by these models. The inclusion of these subgrid scale processes in terms of the resolved quantities is referred to as the problem of parametrization. In this talk, it is shown how symmetries can be used to specify forms of parameterizations in a way such that important invariance properties of the grid scale models are retained. The methods we apply are based on techniques of inverse and direct group classification. They are illustrated by constructing various parametrization schemes for the turbulent vorticity transport in the framework of inviscid two-dimensional ideal fluid mechanics.

[1] Bihlo A. and Popovych R. O., Symmetry preserving parameterization schemes, in preparation.

Conformal invariance and partially nonclassical method

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Based on the observation, that applying the classical Lie group method to a partial differential equation (PDE) yields transformations which do not leave the PDE invariant but modify it by a conformal factor, a somewhat different representation of the nonclassical method procedure is developed. This representation contains free parameters which may be chosen either such that the procedure were equivalent to the common nonclassical method or in another way which, in general, makes the method different from the nonclassical one. Such a modified method is named the "partially nonclassical method" as usually some of the free parameters are set to zero. The partially nonclassical method is applied to the flat steady-state boundary layer (BL) equations, which are of considerable importance in physics (nonclassical similarity reductions of the flat steady-state BL equations have been considered in [1] and [2]). While applying the partially nonclassical method, both the general case and the special case where one of the infinitesimal generators associated with an independent variable is identically zero are considered. In the latter case, as distinct from the common nonclassical method, where solution of the equation for a group generator is more complicated than the solution of the original BL equations, applying the partially nonclassical method yields solvable determining equations. New similarity reductions and exact solutions of the BL equations are found.

[1] Burde G.I., New similarity reductions of the steady-state boundary layer equations, *J. Phys. A: Math. Gen.*, 1996, V.29, 1665–1683.

[2] Saccomandi G., A remarkable class of non-classical symmetries of the steady two-dimensional boundary-layer equations, *J. Phys. A: Math. Gen.*, 2004, V.37, 7005–7017.

Three boson interaction Hamiltonian solvable via Krawtchouk polynomials

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We study a class of Hamiltonians that can be solved by Krawtchouk orthogonal polynomials in two discrete variables. It can be applied for solution of truncated three mode states, or optical tritters, i.e. interferometers which consist of three single-mode fibers and an integrated fiber couplers.

Lambda-symmetries of Dynamical Systems, Hamiltonian and Lagrangian equations

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Symmetries and perturbed symmetries of canonical Hamiltonian equations of motion are considered, with special attention to Lambda-symmetries. After a brief survey of the definition and the properties of Lambda-symmetries in the general context of dynamical systems, the notion of “Lambda-constant of motion” for Hamiltonian equations is introduced. If the Hamiltonian problem is derived from a Lambda-invariant Lagrangian, it is shown how the Lagrangian Lambda-invariance can be transferred into the Hamiltonian context and shown that the Hamiltonian equations turn out to be Lambda-symmetric. Finally, the “partial” (Lagrangian) reduction of the Euler-Lagrange equations is compared with the reduction obtained for the corresponding Hamiltonian equations.

Natural curvilinear coordinates in MHD. Flows with constant total pressure

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For the ideal MHD equations we introduce a curvilinear system of coordinates, which is naturally determined by the geometry of the flow. In these coordinates the MHD equations are reduced to a vector wave equation for the mapping determining the coordinates, and to the Cauchy integral for the density. The suggested from of equations is convenient because its every solution gives explicit description of particles' trajectories and magnetic field lines of the flow. We use this form of the equations to describe incompressible plasma flows with constant total pressure. The description of this class of solutions requires separation of variables in a certain nonlinear scalar equation. The stationary case was described in [1], here we observe the more general non-stationary case. Solutions with functional arbitrariness describing various flows in bent canals, in particular, in torus-shaped configurations are obtained.

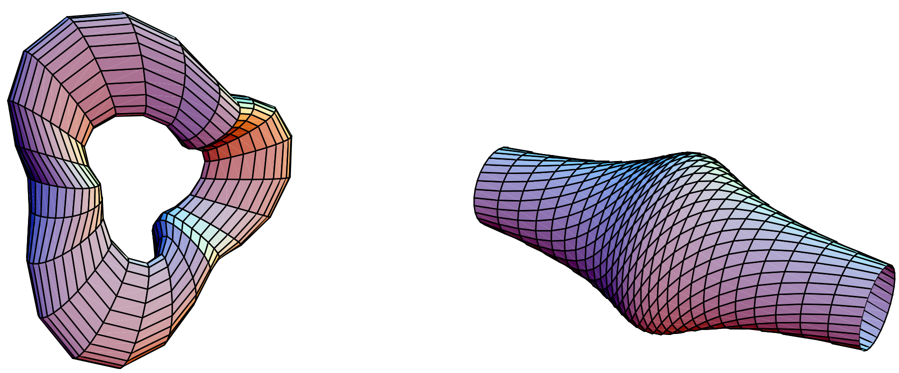

Figure 1. Examples of canals containing plasma flows with constant total pressure

The work was supported by RFBR (project 08-01-00047), Ministry of Science and Education of Russian Federation (project 2.1.1/3543), and by the program of Russian Academy of Sciences (project 14.14.1).

[1] Golovin S.V., Analytical description of stationary ideal MHD flows with constant total pressure, *Phys. Lett. A.*,2010, V.374 901–905.

Poisson-Lie symmetry and solutions of sigma models

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Poisson-Lie T-duality and -plurality was introduced as an extension of the Abelian T-duality to non-Abelian groups of isometries. It is a transformation of nonlinear sigma-models that can relate cases with different geometrical backgrounds. It is based on different decompositions of Drinfel'd doubles, i.e. Lie groups whose algebra is provided with an Ad-invariant bilinear forms. Transformation of classical solutions, boundary conditions, dilaton fields and other properties of sigma models can be done. Examples of interesting sigma models with Poisson-Lie symmetry will be presented.

Two types of discretization of tori of compact simple Lie groups

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For a given compact simply connected simple Lie group G, we describe two types of certain finite sets of lattice points. These sets are subsets of the Lie algebra of the maximal torus of G. The group G and the corresponding affine Weyl group with its even subgroup induce the symmetry of the lattices; their density is set by an arbitrary number M. We review constructions of the lattices and present counting formulas for the numbers of their points. The properties of C-, S- and E-functions on these lattices are discussed. We describe the maximal sets of these pairwise orthogonal functions and calculate Fourier like discrete expansions of arbitrary discrete functions. Application of these transforms to interpolation is presented.

Group analysis of variable coefficient diffusion–convection equations

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In this talk we perform the complete extended group classification for a class of (1+1)-dimensional nonlinear diffusion–convection equations with coefficients depending on the space variable of form $f(x)u_t = (g(x)A(u)u_x)_x + h(x)B(u)u_x$. At first, we construct the usual equivalence group and the extended one including transformations which are non-local with respect to arbitrary elements. The extended equivalence group has interesting structure since it contains a non-trivial subgroup of non-local gauge equivalence transformations. The complete group classification of the class under consideration is carried out with respect to the extended equivalence group and with respect to the set of all point transformations. Usage of extended equivalence and correct choice of gauges of arbitrary elements play the major role for simple and clear formulation of the final results. The set of admissible transformations of this class is preliminary investigated.

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Variational Lie algebroids

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The Lie algebroids over smooth manifolds are a convenient and important construction in geometry, e.g., in Poisson dynamics. We define the variational Lie algebroids over the infinite jet spaces of mappings between smooth manifolds (e.g., from strings to space-time) and give model examples of this construction. We expect it to be relevant in the description of the short-range fundamental interactions of elementary particles. [This is a joint talk with J.W.van de Leur (Utrecht).]

On realizations of Lie algebras of Poincaré group and new Poincaré-invariant equations

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In this talk we consider the problem of construction of the most general partial differential equation that admits as invariance group some known group of local transformations. It is evidently that complete solution of this problem for a certain group of local transformations implies existence of realizations of its Lie algebra in the class of Lie vector fields.

The main problem under consideration in this report is to describe various realizations of Lie algebras of Poincaré groups $P(1,1)$, $P(1,2)$, and their natural generalizations. All that such realizations are considered in the class of Lie vector fields in a space of several independent variables and one dependent variable. Up to diffeomorphism of corresponding spaces the complete lists of realizations of these algebras are obtained.

Then the obtained realizations are used for construction of new Poincaré-invariant second order partial differential equations.

Hermitian quasi-exactly and exactly solvable matrix Schrodinger operators

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The problem of construction of quasi-exactly and exactly solvable one-dimensional stationary Schrödinger models

$$\hat{H}\psi(x) = \left(\partial_x^2 + V(x)\right)\psi(x) = E_k\psi(x) \tag{1}$$

is investigated. Here $V(x)$ is hermitian 2×2 matrix, $\psi(x) = \begin{bmatrix} \psi_1(x) \\ \psi_2(x) \end{bmatrix}$ is two-component spinor (see $[1,2]$). In the papers $[3,4]$ we have extended the Turbiner-Shifman approach to the construction of quasi-exactly solvable (QES) models on line for the case of matrix Hamiltonians. In this way six multi-parameter families of Hermitian quasi-exactly solvable matrix Schrödinger operators (1) were constructed.

In the papers $[5,6]$ our method was further generalized to the construction of exactly solvable (ES) models. Namely, a set of operators used under the construction of Hamiltonians not necessarily generates Lie algebra. Five multi-parameter families of Hermitian exactly solvable matrix Schrodinger operators (1) was constructed.

[1] Shifman M.A., Turbiner A.V., Quantal problems with partial algebraization of the spectrum, *Commun. Math. Phys.*, 1998, V.126, 347–365.
[2] Ushveridze A.G., *Quasi-Exactly Solvable Models in Quantum Mechanics*, Institute of Physics Publishing, Bristol, 1994.
[3] Zhdanov R.Z., On algebraic classification of quasi-exactly solvable matrix models, *J. Phys. A: Math. Gen.*, 1997, V.30, 8761–8770.
[4] Spichak S.V., Zhdanov R.Z., On algebraic classification of Hermitian quasi-exactly solvable matrix Schrodinger operators on line, *J. Phys. A.: Math. Gen.*, 1999, V.32, 3815–3831.
[5] Abramenko A.O., Spichak S.V., On new classes of Hermitian exactly solvable matrix Schrodinger operators, *Mat. Stud.*, 2001, V.15, 44–56 (in Ukrainian).
[6] Lahno V.I., Spichak S.V., Stognii V.I., *Symmetry Analysis of Evolution Type Equations*, Moscow–Izhevsk, RCD, 2004 (in Russian).

Orthogonal polynomials of symmetric and antisymmetric generalizations of trigonometric functions

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The polynomials are built from the new special functions which generalize common sine and cosine functions as $n \times n$ determinants and permanents of the products of the common functions depending on different variables. The polynomials are eigenfunctions of differential operators which will be introduced. Method of construction of the polynomials, their continuous orthogonality and a few identities will be presented. The examples of two dimensional polynomials will be shown.

[1] Hrivnak J., Motlochova L. and Patera J., Two-dimensional symmetric and antisymmetric generalizations of sine functions, *J. Math. Phys*, to appear (arXiv:0912.0241v1).

[2] Hrivnak J. and J. Patera, Two-dimensional symmetric and antisymmetric generalizations of exponential and cosine functions, *J. Math. Phys.*, 2010, V.51, 023515 (arXiv:0911.4209v1).

[3] Motlochova L. and Patera J., Orthogonal polynomials of symmetric and antisymmetric generalizations of trigonometric functions of two variables, in preparation.

Orthogonal polynomials of compact simple Lie groups

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Recursive construction of two infinite families of polynomial of n variables is demonstrated as a uniform method applicable to every semisimple Lie group of rank n. Its result recognizes Chebyshev polynomials of the first and second kind as the special case of the simple group of type A_1. It is done by a substitution of variables into the known functions formed by summing up exponential terms over orbits of the Weyl groups of the Lie group. Real variables are replaced by the lowest orbit functions as the new variables. Basic relation between the polynomials of two families is a direct consequence of the Weyl character formula. Properties of the polynomials follow from the corresponding properties of the orbit functions, namely the orthogonality and discretization. Recurrence relations are shown for the Lie groups of types A_1, A_2, A_3, C_2, C_3, G_2, and B_3 together with a few lowest polynomials.

Group analysis and exact solutions for equations of axion electrodynamics

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The group analysis of systems of non-linear equations which model a generalized axion electrodynamics is carried out. Using the Inönü-Wigner contraction the non-relativistic limit of equations of the standard axion electrodynamics is found. An extended class of exact solutions for the electromagnetic and axionic fields is presented. It is shown that in spite of the manifest relativistic invariance of the theory such solutions can describe propagation with velocities faster than the velocity of light.

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Generalized derivations, twisted cocycles and invariant functions of Lie algebras

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Using suitable generalization of cohomology cocycles we define invariant functions for finite-dimensional Lie algebras. Two of these functions provide complete classification of all complex Lie algebras up to dimension four. We also show application of these invariant functions on one-parametric continua of eight-dimensional nilpotent Lie algebras.

The unreasonable effectiveness of Lie symmetries

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Fifty years ago Eugene P. Wigner published a paper where the effectiveness of Mathematics in the Natural Sciences was generally praised. Following in his footsteps, and to honor the fiftieth anniversary of that publication we show by various examples the "unreasonable" effectiveness of Lie symmetries.

Moving frames, symmetries, and reduction of exterior differential systems

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Continuous pseudogroups appear as the infinite dimensional counterparts of local Lie groups of transformations in various physical and geometrical contexts, including gauge theories, Hamiltonian mechanics and symplectic and Poisson geometry, conformal field theory, symmetry groups partial differential equations, such as the Navier-Stokes and Kadomtsev-Petviashvili equations of fluid mechanics and plasma physics, image recognition, and geometric numerical integration.

In this talk I will discuss a novel reduction algorithm for identifying integral manifolds of exterior differential systems invariant under the action of a continuous pseudogroup, which incorporates my recent joint work with Peter Olver on extending the classical moving frames method to the infinite dimensional situation. With the help of a moving frame, the exterior differential system gives rise to a reduced system on a given cross section to the group action. All integral manifolds of the original system can then be reconstructed from those of the reduced system by the way of an equation of generalized Lie type for the symmetry group, resulting in a two-step algorithm for finding integral manifolds akin to Vessiot's method of group foliation. As examples, I will discuss applications of the reduction process to the construction of analytic solutions to systems of partial differential equations.

Conservation laws and normal forms of evolution equations

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Local conservation laws for evolution equations in two independent variables are studied. In particular, we present normal forms for the equations admitting one or two low-order conservation laws. Examples include Harry Dym equation, Korteweg–de Vries-type equations, and Schwarzian KdV equation. It is also shown that for linear evolution equations all their conservation laws are (modulo trivial conserved vectors) at most quadratic in the dependent variable and its derivatives.

Consider an evolution equation in two independent variables,

$$u_t = F(t, x, u_0, u_1, \dots, u_n), \quad n \geqslant 2, \quad F_{u_n} \neq 0, \tag{1}$$

where $u_j \equiv \partial^j u/\partial x^j$, $u_0 \equiv u$, and $F_{u_j} = \partial F/\partial u_j$. By $\mathrm{ord_d}\mathcal{L}$ we denote the density order of a conservation law $\mathcal{L}$ of (1).

Theorem 1. *Equation* (1) *admits a nonzero conservation law* $\mathcal{L}$ *with* $\mathrm{ord_d}\mathcal{L} \leqslant 1$ *(resp.* $\mathrm{ord_d}\mathcal{L} = 0$*) iff it is locally reduced by a contact (resp. point) transformation to the form*

$$\tilde u_{\tilde t} = D_{\tilde x} G(\tilde t, \tilde x, \tilde u_0, \dots, \tilde u_{n-1}), \quad G_{\tilde u_{n-1}} \neq 0. \tag{2}$$

Corollary 1. *If equation* (1) *with* $n \geqslant 4$ *(resp.* $n \geqslant 5$*) has two linearly independent conservation laws* $\mathcal{L}^{\mathrm{I}}$ *and* $\mathcal{L}^{\mathrm{II}}$*, where* $\mathrm{ord_d}\mathcal{L}^{\mathrm{I}} \leqslant 1$ *and* $\mathrm{ord_d}\mathcal{L}^{\mathrm{II}} \leqslant n/2 - 1$ *(resp.* $\mathrm{ord_d}\mathcal{L}^{\mathrm{II}} < n/2 - 1$*) then it can be locally reduced by a contact transformation to the form* (2) *where* G *is linear fractional (resp. linear) with respect to* $\tilde u_{n-1}$*. If* $\mathrm{ord_d}\mathcal{L}^{\mathrm{I}} = 0$ *then the contact transformation in question is a prolongation of a point transformation.*

Theorem 2. *Equation* (1) *has (at least) two linearly independent conservation laws of density order 0 iff it can be locally reduced by a point transformation to the form*

$$\tilde u_{\tilde t} = D_{\tilde x}^2 H(\tilde t, \tilde x, \tilde u_0, \dots, \tilde u_{n-2}), \quad H_{\tilde u_{n-2}} \neq 0. \tag{3}$$

Corollary 2. *If equation* (1) *with* $n \geqslant 5$ *(resp.* $2 \leqslant n \leqslant 4$*) has two linearly independent conservation laws* $\mathcal{L}^{\mathrm{I}}$ *and* $\mathcal{L}^{\mathrm{II}}$ *with* $\mathrm{ord_d}\mathcal{L}^{\mathrm{I}} \leqslant 1$ *and* $\mathrm{ord_d}\mathcal{L}^{\mathrm{II}} < n/2 - 1$ *(resp.* $\mathrm{ord_d}\mathcal{L}^{\mathrm{I}} = \mathrm{ord_d}\mathcal{L}^{\mathrm{II}} = 0$*), then the right-hand side* F *of* (1) *has the form*

$$F = F_3 u_n + F_2 u_{n-1}^2 + F_1 u_{n-1} + F_0,$$

where $F_0, \dots, F_3$ *are differential functions of order less than* $n-1$*.*

Theorem 3. *For any linear* $(1+1)$*-dimensional evolution equation of even order its space of conservation laws is exhausted by linear ones and is isomorphic to the solution space of the corresponding adjoint equation.*

Theorem 4. *For any linear* $(1+1)$*-dimensional evolution equation of odd order, the space of its conservation laws is spanned by linear and quadratic ones.*

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Comparing three dimensional alternating, symmetric and antisymmetric interpolations

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New class of special functions in 3D, based on the permutation group S_3 and its subgroups, are described and used for Fourier-like expansion of digital data given on lattice of any density. Continuous interpolation of the data is studied. It is shown that its quality is strongly influenced by shifting the lattice that carryies the data, with respect to the original group-defined position of the lattice. Two examples of such interpolations are presented.

Potential symmetries to nonlinear diffusion equations

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In this talk, the potential symmetry method is developed to study the systems of nonlinear diffusion equations and higher-order nonlinear evolution equations. For the systems of nonlinear diffusion equations, potential variables are introduced through conservation laws. Such conservation laws yield equivalent systems of PDEs with the given dependent and potential variables as dependent variables. Lie point symmetries of the auxiliary system yield potential symmetries of the given systems. For a class of fourth-order evolution equations, using the result of classification on contact symmetries of evolution equations due to Sokolov and a approach due to Zhdanov, we obtain all inequivalent fourth-order evolution equations admitting potential symmetries.

Variational principles in control and template matching

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Reduction of variational principles is presented. Applications to control theory where the dynamics is determined by the symmetry Lie algebra is discussed. It is shown that the control satisfies the motion equations of a reduced variational principle. As another application, template matching in computational anatomy is presented.

The pulsrodon in 2+1-dimensional magnetogasdynamics and Hamiltonian Ermakov reduction

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An elliptic vortex-type ansatz introduced into a 2+1-dimensional system governing rotating homentropic magnetogasdynamics with a parabolic gas law is shown to lead to an eight-dimensional nonlinear dynamical system which admits exact analytic solution in terms of an elliptic integral representation.A novel magnetogasdynamic analogue of the pulsrodon of shallow water f-plane theory is isolated thereby. In the case of a purely transverse magnetic field,the general dynamical system is shown to have underlying Hamiltonian structure of Ermakov-type. Pulsrodon-type solutions may again be constructed.

Symmetries and reductions for the Camassa Holm hierarchy in 2+1 dim multisoliton solutions

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The non isospectral problem (Lax pair) associated with a hierarchy in 2+1 dimensinos that generalizes the well known Camassa Holm hierarchy is presented. Here, we have investigated the non-classical symmetries of this Lax pair when the spectral parameter is considered a field. These symmetries can be written in terms of five arbitrary constants and three arbitrary functions. Different similarity reductions associated with these symmetries have been derived. Of particular interest are the reduced hierarchies whose 1+1 Lax pair is also non isospectral

Symmetries and invariant solutions of PDEs on superspace

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The generalization of the symmetry analysis to partial differential equations on superspace will be reviewed. That means

1. how one can find the Lie superalgebra of infinitesimal symmetries of a given PDE on superspace,
2. whether and how one can construct invariant solutions involving anticommuting variables.

The application of these methods to supersymmetric nonlinear wave equations, e.g. supersymmetric sine-Gordon, sinh-Gordon and polynomial Klein-Gordon equations, will be outlined and the arising difficulties discussed.

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Lie symmetries and certain equations of Financial Mathematics

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We explore the application of symmetry in the sense of Lie's Theory to the algebraic resolution of evolution partial differential equations for problems arising in Financial Mathematics and demonstrate that many problems are susceptible an algorithmic treatment. In particular we show how certain equations which have been solved by *ad hoc* methods are easily solved using the algebraic approach. Some equations considered are the Cox–Ingersoll–Ross equation with time-dependent parameters and the Heston Problem of Stochastic Volatility.

Acknowledgements. This work is part of a project funded by the Research Promotion Foundation of Cyprus, Grant Number ΠΡΟΣΕΛΚΥΣΗ ΠΡΟΕΜ/0308/02, devoted to the algebraic resolution of nonlinear partial differential equations.

Equations of vibrational convection in binary mixture: symmetry properties and exact solutions

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Vibrational convection refers to specific flows, which appear in a fluid with density inhomogeneity under periodical external force. The variations of density can be caused by the thermal or compositional gradient (if a mixture with two or higher numbers of components is considered). One usually speaks about high–frequency vibration if the period of vibration is much smaller than the characteristic times of the system (viscous, thermal, and diffusion times). In this case, the velocity, pressure, temperature, and concentration fields can be represented as a sum of 'fast' component, which oscillates with the external frequency, and 'slow' component, which is obtained by time averaging of a related quantity [1]. The averaged flows induced by external vibration can appear in the absence of gravity and cause significant heat and mass transfer. Theoretical and experimental study of thermovibrational convection in microgravity has been recently performed in [2, 3].

In this work, the model of averaged motion in a binary mixture with the Soret effect is considered. The difference between this model and classical Navier–Stokes and heat/mass trasnfer equations lies in an additional equation for the amlitude of pressure oscillations and the presence of averaged vibrational force term in the momentum equation. We have performed symmetry classification of this model with respect to the constant parameters (vibration vector, gravity vector, thermal and solutal expansion coefficients, thermal diffusivity, and diffusion and thermal diffusion coefficients).

The group–theoretical interpretation of existing solutions [1] is given. A new partially invariant solution is constructed and applied to describing the separation of binary mixture in a thermogravitational column under high–frequency vibration (thermogravitational column is an experimental apparatus for separating mixtures and measruing transport coefficients [4]). The column is represented by a vertical layer between two flat plates with different temperatures. The horizontal separation due to the Soret effect combined with vertical convective current leads to the separation of species in vertical direction. We have found the distributions of velocity, temperature, and concentration in the column in the presence of longitudinal vibrations. It is shown that such vibrations lead to the increase of convective velocity and decrease of vertical separation. At the same time, the horizontal separation is not affected by vibrations.

This work is supported by the Russian President Grant MK-299.2009.1.

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[2] Mialdun A., Ryzhkov I.I., Melnikov D.E., and Shevtsova V., Experimental evidence of thermal vibrational convection in a nonuniformly heated fluid in a reduced gravity environment, *Phys. Rev. Let.*, 2008, V.101, 084501.

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On the integrability of Hirota-Kimura discretizations

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We discuss mysterious properties of Hirota-Kimura (bilinear) discretization of integrable systems with quadratic vector fields. This discretization scheme will be shown to tend to preserve integrability, while the mathematical reasons of this phenomenon are not yet completely understood.

Fundamental solution in classical elasticity via group analysis

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The observation that the fundamental solutions of the linear partial differential equations of mathematical physics are often group-invariant solutions [4] led to the development of a systematic group theoretic approach to fundamental solutions. This new approach combines the philosophy of Lie symmetries with the theory of distributions (generalized functions) [4].

Here we extend the approach suggested in [4] to compute the fundamental solution of the system that describes the static equilibrium in classical elasticity of an isotropic and homogeneous medium under no body forces. The equations of static equilibrium in classical elasticity are obtained from the divergence of the stress tensor. In linear elasticity the components of the stress tensor are related linearly to the components of the strain tensor which are related linearly to derivatives of the displacement components.

The explicit expressions of fundamental solutions in the theory of elasticity were obtained in different way in the literature [1–3, 5].

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Localized travelling wave solutions to the time-delayed convection-reaction-diffusion equation

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There is number of papers dealing with the asymptotic and attracting features of self-similar solutions to parabolic evolutionary equations. It is established that in those cases when the initial data are chosen among the integrable ("finite-mass") functions, a common feature of the solutions of the Cauchy problems to the nonlinear diffusion as well as the nonlinear convection-diffusion equation, is the asymptotic tendency to the Gauss-like bell-shaped solution, quickly vanishing, as the time passes by [1]. Somewhat different behavior is reported to demonstrate the hyperbolic generalizations to Burgers equation. Depending on the amplitude of the "finite-mass" initial data, solutions of the Cauchy data either tend asymptotically to quickly disappearing Gauss-like solutions, or collapse in finite time [2]. It is rather well-recognized that asymptotic vanishing of the "finite-mass" transport equations' solutions is due to the presence of dissipation, while the collapses are attributed to the presence of unbalanced instability. Previous investigations evidence that the dissipative-type equations possess stable nonvanishing patterns, if some energy source is incorporated into the model. In this work we consider a sort of such model, referred to as the hyperbolic generalization of the reaction-diffusion-convection equation. Our aim is to show the existence of wide variety of localized TW solutions such as solitary waves, compactons, shock fronts and cuspnos, evolving without change of their shape.

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Reduction of multidimensional wave equations to two-dimensional equations: investigation of possible reduced equations

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We study possible Lie and non-classical reductions of multidimensional wave equations and the special classes of possible reduced equations — their symmetries and equivalence classes. Such investigation allowed to find many new conditional and hidden symmetries of the original equations.

Instructions for Paper Submission

All papers should be written in good English and should be no longer than 20 pages. The files of the papers shall be prepared in the LaTeX2ϵ format using the style file that can be found in the web page of the workshop. Amendments of the style file are not allowed. Abbreviations for standard LaTeX commands are not allowed in the paper!

Please adhere to the following order of presentation:Article title, Author(s), Affiliation(s), E-mail(s), Abstract, Main text, Acknowledgements, Appendices, References.

References to other work should be consecutively numbered in the text using square brackets and listed by number in the Reference list. References to books should include the author's name; year of publication; title; page numbers where appropriate; publisher; place of publication, in the order given in the example below.

Olver P., *Applications of Lie Groups to Differential Equations*, Springer-Verlag, New York, 1986.

References to articles in conference proceedings should include the author's name; year of publication; article title; editor's name (if any); title of proceedings; first and last page numbers, in the order given in the example below.

Ivanova N.M., Popovych R.O. and Sophocleous C., Conservation laws of variable coefficient diffusion–convection equations, 2005, *Proceedings of Tenth International Conference in Modern Group Analysis (Larnaca, Cyprus, 2004)*, 107–113.

References to articles in periodicals should include the author's name; year of publication; article title; full title of periodical; volume number; first and last page numbers, in the order given in the example below.

Nikitin A.G., Group classification of systems of non-linear reaction-diffusion equations with general diffusion matrix. I. Generalized Ginzburg-Landau equations *J. Math. Anal. Appl.*, 2006, V.324, 615–628.

A sample paper can be found in the web page of the workshop.

Contents

Notes

Notes

Notes