Relativistic particle of arbitrary spin in the Coulomb and magnetic-monopole field

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Exact solutions of relativistic wave equations for any spin charged particle in the Coulomb and magnetic-monopole fields are found.

1. Introduction

The description of interaction of charged spinning particle with external field is important problem of quantum mechanics. The interest in such problems is stimulated by the research of quark models with effective potential (see, e.q., [1]).

The experimental discovery of the relatively stable resonances with spins $s > \frac{1}{2}$ and searches of the exotic atoms, in which these resonances play the role of an orbital particles [2, 3] lead to the necessity of description of high-spin particle motion in an external field. At the same time relativistic wave equations for such particles lead to contradictions of principle — such as the absence of stable solutions in Coulomb field [4], the causality violation [5], etc. (see, e.q., [6]).

In papers [7, 8] Poincaré-invariant wave equations for particles of arbitrary spin are proposed which allow us to avoid many of these difficulties. By using these equations the solutions of many problems connected with any spin particle motion in an external field have been found for homogeneous magnetic field, Coulomb one and also for Redmond field — i.e. the combination of plane wave and homogeneous magnetic, field [9, 10]. In [11] the alternative possibility, of describing the spinning particle in the Coulomb field is considered, one that makes use of Galilei-invariant wave equations.

In present paper the problem of interaction of any spin relativistic particle with magnetic-monopole field is solved, using the equations proposed in [7, 8]. Such a problem for spinless particle was first considered by Dirac [12] and Tamm [13]. Harish-Chandra [14] obtained the exact solution of Dirac equation for electron interacting with magnetic-monopole field. A number of publications, devoted to the description of the motion of a charge in monopole field has appeared last time (see, e.g., [15–18]), but the case of a particle of any spin was not yet considered.

Besides we obtain the exact solutions of Poincaré-invariant equations for particles with arbitrary spin, interacting with the combination of the Coulomb and magnetic-pole fields.

2. Poincaré-invariant equations for particles of arbitrary spin

We will start from the following equations, describing the relativistic particle of spin s in an external electromagnetic field [7, 8]:

$$\begin{gathered}\left[\Gamma_\mu\pi^\mu - m + \frac{e}{4m}(1-i\Gamma_4)\left(\frac{1}{s}S_{\mu\nu} - i\Gamma_\mu\Gamma_\nu\right)F^{\mu\nu}\right]\Psi = 0,\\ (\Gamma_\mu\pi^\mu + m)(1-i\Gamma_4)[S_{\mu\nu}S^{\mu\nu} - 2s(s-1)]\Psi = 16ms\Psi,\end{gathered} \tag{2.1}$$

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where $\Psi = \Psi(x)$ is the $8s$-component wave function, $x = (x_0, x_1, x_2, x_3)$, $\pi_\mu = -i(\partial/\partial x^\mu) - eA_\mu$, A_μ is the vector potential, $F^{\mu\nu}$ is the electromagnetic-field tensor, Γ_μ are $(8s \times 8s)$-dimensional matrices satisfying the Clifford algebra

$$\Gamma_\mu\Gamma_\nu + \Gamma_\nu\Gamma_\mu = 2g_{\mu\nu}, \tag{2.2}$$

$\Gamma_4 = \Gamma_0\Gamma_1\Gamma_2\Gamma_3$, $S_{\mu\nu}$ are the generators of the representation $\left[D\left(\frac{1}{2},0\right)\otimes D\left(0,\frac{1}{2}\right)\right]\otimes D\left(s-\frac{1}{2},0\right)$ of Lorentz group.

In the case $s=\frac{1}{2}$ the system (2.1) is reduced to Dirac equation for electron. If s is arbitrary integer or half-integer, this system describes the causal motion of the charged particle of spin s in an external electromagnetic field [7, 8].

To solve the above-mentioned problems it is convenient to pass from eqs.(2.1) to the system of second-order equations. Multiplying eqs.(2.1) from the left by $\frac{1}{2}(1\pm i\Gamma_4)$ and expressing $\Psi_+ = \frac{1}{2}(1+i\Gamma_4)\Psi$ via $\Psi_- = \frac{1}{2}(1-i\Gamma_4)\Psi$ we obtain

$$\left(\pi_\mu\pi^\mu - m^2 - \frac{e}{2s}S_{\mu\nu}F^{\mu\nu}\right)\Psi_- = 0, \tag{2.3a}$$

$$[S_{\mu\nu}S^{\mu\nu} - 4s(s+1)]\Psi_- = 0, \tag{2.3b}$$

$$\Psi_+ = \frac{1}{m}\Gamma_\mu\pi^\mu\Psi_-. \tag{2.3c}$$

According to (2.3), solving of eqs.(2.1) is reduced to finding of the function Ψ_-, satisfying (2.3a), (2.3b) inasmuch as general solution of eqs.(2.1) may be presented as $\Psi \equiv \Psi_+ + \Psi_-$, and Ψ_- is expressed via Ψ_+ in accordance with (2.3c).

It follows from (2.3b) that the function Ψ_- has only $2s+1$ nonzero components and is spinor from the space of $D(s,0)$ representation of the Lorentz group. On the set of such functions the matrices $S_{\mu\nu}$ are reduced to $(2s+1)\times(2s+1)$-dimensional generators of the irreducible representation $D(s)$ of O_3 group (indicated below as $\boldsymbol{S} = (S_1, S_2, S_3)$), and eq.(2.3a) comes to the following form:

$$\left[\pi_\mu\pi^\mu - m^2 - \frac{e}{m}\boldsymbol{S}(\boldsymbol{H} - i\boldsymbol{E})\right]\Phi_s = 0, \tag{2.4}$$

where Φ_s is $(2s+1)$-component function (including nonzero components of Ψ_-), $\boldsymbol{E}$ and $\boldsymbol{H}$ are the vectors of electric and magnetic fields, respectively. For $s=\frac{1}{2}$ eq.(2.4) coincides with well-known Zaiteev–Feynman–Gell-Mann equation [19, 20].

3. Arbitrary-spin particle in the magnetic-monopole field

In the spherical co-ordinates the vector potential and the corresponding vectors of the electric and magnetic-field strength created by the magnetic monopole are [14]

$$\begin{aligned} &A_0 = A_r = A_\theta = 0, \qquad A_\varphi = \frac{n}{2c}(1-\cos\theta), \\ &\boldsymbol{E} = 0, \qquad \boldsymbol{H} = \frac{n}{2e}\cdot\frac{\boldsymbol{r}}{r^3}, \end{aligned} \tag{3.1}$$

where n is integer, $\boldsymbol{r} = (r\sin\theta\cos\varphi, r\sin\theta\sin\varphi, r\cos\theta)$.

Writing eqs.(2.4), (3.1) in the spherical co-ordinates one obtains

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}\Phi + \frac{1}{r^2}\Delta^*\Phi + (\varepsilon^2 - m^2)\Phi = \frac{n}{2s}\frac{\boldsymbol{S}\cdot\boldsymbol{r}}{r^3}\Phi, \tag{3.2}$$

where ε is the stationary-state energy,

$$\Delta^* = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \frac{in}{1+\cos\theta}\frac{\partial}{\partial\theta} - \frac{n^2}{4}\frac{1-\cos\theta}{1+\cos\theta}.$$

Equation (3.2) may be solved by separation of variables for any value of spin s. With the help of the unitary transformation $\Psi \to \Omega = V\Psi$, where

$$V = \exp[-iL_3\varphi]\exp[i(S_2\cos\varphi - S_1\sin\varphi)\theta], \qquad L_3 = -i\frac{\partial}{\partial\varphi}, \tag{3.3}$$

eq.(3.2) comet to such a form, in which the matrix $\boldsymbol{S}\cdot\boldsymbol{r}/r^3$ on the r.h.s. is diagonal and equal to S_3/r^2:

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}\Omega + \frac{1}{r^2}\Delta'^*\Omega + (\varepsilon^2 - m^2)\Omega = \frac{n}{2s}\frac{1}{r^2}S_3\Omega. \tag{3.4}$$

Here

$$\begin{aligned}\Delta'^* = {}& K^2 - \boldsymbol{S}^2 - 2S_3^2 + nS_2 + \frac{n^2}{4} + 2iS_2(1-w^2)^{1/2}\frac{d}{dw} - \\ & -2S_1\frac{1}{(1-w^2)^{1/2}}\left[L_3 + \left(\frac{n}{2}+S_3\right) - \left(\frac{n}{2}+S_3\right)w\right],\end{aligned} \tag{3.5}$$

where

$$\begin{aligned}K^2 = {}& (1-w^2)\frac{d^2}{dw^2} - 2w\frac{d}{dw} - \\ & -\frac{[L_3 + (n/2+S_3) - (n/2+S_3)w]^2}{1-w^2} - \left(\frac{n}{2}+S_3\right)^2, \\ w = {}& \cos\theta.\end{aligned} \tag{3.6}$$

The solutions of eq.(3.4) can be represented as an expansion in Jacobi polinomials [14] and eigenfunctions of the operator L_3

$$\Omega = \sum_\sigma F_\sigma(r)P^k_{n/2+j,n/2+\sigma}(w)\exp[-i(j-\sigma)\varphi], \tag{3.7}$$

where $P^k_{n/2+j,n/2+\sigma}$ is the complete set of the normalized eigenfunctions of the commuting operators $J_3 = L_3 + S_3$, S_3 and K^2 (3.6) which correspond to the eigenvalues j, σ and $-k(k+1)$, respectively, moreover

$$\begin{aligned}& k \geq \left|\tfrac{n}{2}+\sigma\right|, \qquad \left|\tfrac{n}{2}+j\right| \quad \text{and} \quad k - \left(\tfrac{n}{2}+\sigma\right) \quad \text{are integers}, \\ & \sigma = -n_{sk}, -n_{sk}+1, \ldots, n_{sk}, \qquad n_{sk} = \min(s,k).\end{aligned} \tag{3.8}$$

Using recurrent relations [14]

$$\begin{aligned}& \left\{(1-w^2)^{1/2}\frac{d}{dw} + \frac{\nu' - \mu' w}{(1-w^2)^{1/2}}\right\}P^k_{\nu'\mu'}(w) = \\ & \qquad = [(k+\mu')(k-\mu'+1)]^{1/2}P^k_{\nu'\mu'-1}(w),\end{aligned} \tag{3.9a}$$

$$\left\{(1-w^2)^{1/2}\frac{d}{dw}-\frac{\nu'-\mu' w}{(1-w^2)^{1/2}}\right\}P^k_{\nu'\mu'}(w)= \\ =-[(k-\mu')(k+\mu'+1)]^{1/2}P^k_{\nu'\mu'+1}(w), \tag{3.9b}$$

and formulae for the matrices S_1 and S_2 in Gel'fand–Zeytlin basis [21], we come to the following equations for radial function $F_\sigma(w)$:

$$DF_\sigma(r)=r^{-2}A_{\sigma\sigma'}F_{\sigma'}(r), \tag{3.10}$$

where

$$D=(\varepsilon^2-m^2)+\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}-\frac{k(k+1)-n^2/4}{r^2}, \tag{3.11}$$

$$A_{\sigma\sigma'}=\left[s(s+1)-2\sigma^2+\frac{1-2s}{2s}n\sigma\right]\delta_{\sigma\sigma'}-\Lambda_{\sigma\sigma'}, \\ \Lambda_{\sigma\sigma'}=0, \qquad \sigma'\neq\sigma\pm1, \\ \Lambda_{\sigma\sigma-1}=\Lambda_{\sigma-1\sigma}=-\left[(s+\sigma)(s-\sigma+1)\left(k+\frac{n}{2}+\sigma\right)\left(k-\frac{n}{2}-\sigma+1\right)\right]^{1/2}. \tag{3.12}$$

Since the matrix $A_{\sigma\sigma'}$ is diagonalizable, system (3.10) can be reduced to the system of noncoupled equations

$$D\hat{F}_\sigma(r)=r^{-2}B^{sk}_\sigma\hat{F}_\sigma(r), \qquad \hat{F}_\sigma=u_{\sigma\sigma'}F_{\sigma'}, \tag{3.13}$$

where B^{sk}_σ are the matrix $A_{\sigma\sigma'}$ eigenvalues, which coincide with the roots of the characteristic equation

$$\det\|A_{\sigma\sigma'}-B^{sk}_\sigma\delta_{\sigma\sigma'}\|=0, \tag{3.14}$$

$u_{\sigma\sigma'}$ is the operator diagonalizing the matrix $A_{\sigma\sigma'}$.

Each of eqs.(3.13) by the replacement of the variable $\rho=(\varepsilon^2-m^2)^{1/2}r$ reduces to the wen-known one [14]

$$\frac{d^2\hat{F}}{d\rho^2}+\frac{2}{\rho}\frac{d\hat{F}}{d\rho}+\left[1-\frac{k(k+1)-n^2/4+B^{sk}_\sigma}{\rho^2}\right]\hat{F}=0, \tag{3.15}$$

the solution of which (limited at the point $\rho=0$) is expressed via Bessel's function

$$\hat{F}=\frac{1}{\sqrt{\rho}}J_{\sqrt{(k+n/2+1/2)(k-n/2+1/2)+B^{sk}_\sigma}}(\rho), \tag{3.16}$$

where k satisfies of the conditions (3.8).

One can make sure by the direct verification that at least for $s<\frac{3}{2}$, $(k+n/2+1/2)(k-n/2+1/2)+B^{sk}_\sigma>0$. This means that $\varepsilon>m$, and so particle with spin $s<\frac{3}{2}$ in magnetic-pole field has continuous energy spectrum and has not coupled states. In the cases $s=0$ and $s=\frac{1}{2}$ the absence of coupled states was demonstrated by Dirac [12] and Harish-Chandra [14].

According to the above the explicit solution of the wave eq.(2.4) for the case in which the external field source is a magnetic monopole has the form

$$\Phi_s(t,\boldsymbol{r}) = \frac{N}{\sqrt{(\varepsilon^2-m^2)^{1/2} r}} \exp[-i\varepsilon t] \exp[i(S_2\cos\varphi - S_1\sin\varphi)\theta]\times$$
$$\times \exp[-i(j-\sigma)\varphi] u^{-1}_{\sigma\sigma} P^{k}_{n/2+j,n/2+\sigma'}(\theta) J_{\sqrt{(k+n/2+1/2)(k-n/2+1/2)+B^{sk}_{\sigma'}}}(\sqrt{\varepsilon^2-m^2}\cdot r), \tag{3.17}$$

where N is the normalization constant. Solutions of the starting system (2.1) may be expressed through the function (3.17) with the help of the relations (2.3c).

Let us give the explicit expressions for B^{sk}_{σ} and $u^{sk}_{\sigma\sigma'}$, if $s \le 1$:

$$B^{\frac{1}{2}k}_{\pm\frac{1}{2}} = \frac{1}{4} \pm \left[\left(k+\frac{n}{2}+\frac{1}{2}\right)\left(k-\frac{n}{2}+\frac{1}{2}\right)\right]^{1/2},$$

$$B^{1k}_{\sigma} = 2\sqrt{-p/3}\cos[(\gamma+\sigma\pi)/3],$$

$$\cos\gamma = \frac{q}{2\sqrt{-(p/3)^2}}, \qquad p = -(2k+1)^2 + \frac{3}{4}n^2 - \frac{1}{3},$$

$$q = -\frac{8}{3}k(k+1) - \frac{16}{27}, \qquad \sigma = 0, \pm 1;$$

$$u^{\frac{1}{2}k}_{\sigma\sigma'} = \begin{pmatrix} c_1 & -c_1 \\ c_2 & c_2 \end{pmatrix}, \qquad \sigma,\sigma' = \pm\frac{1}{2};$$

$$u^{1k}_{\sigma\sigma'} = \begin{pmatrix} \dfrac{p_1}{n/2+B^{1k}_1}\beta_1 & \beta_1 & \dfrac{p_2}{B^{1k}_1-n/2}\beta_1 \\ \dfrac{p_1}{n/2+B^{1k}_0}\beta_2 & \beta_2 & \dfrac{p_2}{B^{1k}_0-n/2}\beta_2 \\ \dfrac{p_1}{n/2+B^{1k}_{-1}}\beta_3 & \beta_3 & \dfrac{p_2}{B^{1k}_{-1}-n/2}\beta_3 \end{pmatrix}, \quad \sigma,\sigma' = 0,\pm 1;$$

$$p_1 = -\left[2\left(k-\frac{n}{2}\right)\left(k+\frac{n}{2}+\frac{1}{2}\right)\right]^{1/2}, \quad p_2 = -\left[2\left(k+\frac{n}{2}\right)\left(k-\frac{n}{2}+\frac{1}{2}\right)\right]^{1/2}.$$

Here c_1, c_2, β_1, β_2, β_3 are arbitrary nonzero constants.

4. Arbitrary spin particle in the Coulomb field

In the case of Coulomb potential $A_0 = Ze/r$, $\boldsymbol{A} = 0$, eq.(2.4) in spherical coordinates takes the following form:

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}\Phi + \frac{1}{r^2}\Delta\Phi + \left[\left(\varepsilon + \frac{\alpha}{r}\right)^2 - m^2\right]\Phi = -\frac{i\alpha}{sr^3}(\boldsymbol{S}\cdot\boldsymbol{r})\Phi, \tag{4.1}$$

where $\alpha = Ze^2$, Δ is an angular part of Laplace operator.

Equation (4.1), as eq.(3.2), has exact solutions in separated variables. In [9] eq.(4.1) is solved by using of the spherical spinor basis. Here we shall obtain the expressions of eq.(4.1) solutions through Jacobi polinomials which are more convenient basis in more general case of combination of the Coulomb and magnetic-pole potentials.

In such a way, as was done in previous section, we shall pass to the representation, in which the matrix $\boldsymbol{S}\cdot\boldsymbol{r}/r^3$ is diagonal. Using for this purpose the transformation operator (3.3), we obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}\Omega+\frac{1}{r^2}\Delta'^{*}_{n=0}\Omega+\left[\left(\varepsilon+\frac{\alpha}{r}\right)^2-m^2\right]\Omega=-\frac{i\alpha}{sr^2}S_3\Omega, \tag{4.2}$$

where $\Delta'^{*}_{n=0}$ is the operator (3.5) with $n=0$.

Representing the solutions of eq.(4.2) in the form (3.7) one comes to eq.(3.10) for the radial wave function, where

$$D=\left(\varepsilon+\frac{\alpha}{r}\right)^2-m^2+\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}-\frac{k(k+1)}{r^2}, \tag{4.3}$$

$$A^{*}_{\sigma\sigma'}=\left[s(s+1)-2\sigma^2-\frac{i\alpha}{s}\sigma\right]\delta_{\sigma\sigma'}-\Lambda^{*}_{\sigma\sigma'}, \tag{4.4}$$

$\Lambda^{*}_{\sigma\sigma'}$ is the matrix (3.12) corresponding to $n=0$.

The matrix $A^{*}_{\sigma\sigma'}$ is diagonalizable, so the system of equations (3.10), (4.3), (4.4) is equivalent to noncoupled eqs.(3.13), (4.3), where B^{sk}_{σ} are the roots of the matrix (4.4) characteristic equation (for explicit expressions for the coefficients B^{sk}_{σ} see [9]). Each of these equations in its turn is reduced to the well-known equation of the for [22]

$$z\frac{d^2y}{dz^2}+\frac{dy}{dz}+\left(\delta-\frac{z}{4}-\frac{l^2}{4z}\right)y=0, \tag{4.5}$$

where

$$\begin{gathered} z=2(m^2-\varepsilon^2)^{1/2}r, \qquad y=\frac{1}{2}\left(\frac{z}{m^2-\varepsilon^2}\right)^{1/2}\hat{F}, \\ \delta=\frac{\varepsilon\alpha}{(m^2-\varepsilon^2)^{1/2}}, \qquad l^2=(2k+1)^2+4(B^{sk}_{\sigma})^2-4\alpha^2. \end{gathered} \tag{4.6}$$

In the case $\varepsilon^2-m^2<0$ (boundary states) the allowed values of δ are

$$\delta=(l+1)/2+n', \qquad n'=0,1,2,\ldots,$$

hence

$$\varepsilon=m\left[1+\frac{\alpha^2}{\left(n'+\frac{1}{2}+\left[\left(k+\frac{1}{2}\right)^2-\alpha^2+B^{sk}_{\sigma}\right]^{1/2}\right)^2}\right]^{-1/2} \tag{4.7}$$

and the solutions of eq.(4.5) are

$$y=\exp[-z/2]z^{l/2}Q^{l}_{n'}(z), \tag{4.8}$$

where $Q^{l}_{n'}$ are Laguerre polynomials [22].

For the continuous spectrum $\varepsilon^2-m^2>0$ the solution of eq.(4.5) limited at the point $z=0$ has the form

$$y=\exp[-i\tau/2]\tau^{l/2}F\left(\frac{l+1}{2}+i\gamma,l+1,i\tau\right), \tag{4.9}$$

where F is a degenerated hypergeometric function, $\tau=-iz$, $\gamma=i\delta$.

So we have obtained the solution of Kepler problem for quantum-mechanical particle of any spin. The energy spectrum of such a particle is determined by formula (4.7) (for coupled states), and radial wave function in representation, where matrix $A^*_{\sigma\sigma'}$ is diagonal, has the form (4.8) or (4.9). The discussion of the spectrum (4.7) is given in [9].

5. Arbitrary spin particle in the combined field

Now we shall consider the motion of a charged particle in central field which is the combination of Coulomb and magnetic monopole ones, when

$$\begin{aligned} A_0 &= \frac{ze}{r}, \qquad \boldsymbol{A} = -\frac{n}{2e}\frac{\boldsymbol{r}\times\boldsymbol{n}}{r(r+\boldsymbol{r}\cdot\boldsymbol{n})}, \\ \boldsymbol{E} &= -\frac{\alpha\boldsymbol{r}}{r^3}, \qquad \boldsymbol{H} = \frac{n}{2e}\frac{\boldsymbol{r}}{r^3}. \end{aligned} \tag{5.1}$$

Writing down the corresponding eq.(2.4) in spherical co-ordinates and representing the solution in the form (3.7), one comes to the radial equation in the form (3.10), where

$$\begin{aligned} D &= \left(\varepsilon+\frac{\alpha}{r}\right)^2 - m^2 + \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{k(k+1)-n^2/4}{r^2}, \\ A_{\sigma\sigma'} &= \left[s(s+1) - 2\sigma^2 + \frac{1-2s}{2s} - \frac{i\alpha}{s}\sigma\right]\delta_{\sigma\sigma'} - \Lambda_{\sigma\sigma'}, \end{aligned} \tag{5.2}$$

and the matrix elements $\Lambda_{\sigma\sigma'}$ are defined by eq.(3.12).

The system (3.10), (5.2) in its turn is reduced to the set of noncoupled eqs.(3.13), where B^{sk}_σ are the matrix (5.2) eigenvalues. So the energy values, corresponding to the coupled states, are

$$\varepsilon = m\left\{1 + \frac{\alpha^2}{\left(n' + \frac{1}{2} + \left[\left(k+\frac{1}{2}\right)^2 + n^2/4 - \alpha^2 + B^{sk}_\sigma\right]^{1/2}\right)^2}\right\}^{-1/2}, \tag{5.3}$$

where $n' = 0, 1, 2, \ldots$, $n = 0, 1, 2, \ldots$, and possible values of k are given in (3.8).

Formula (5.3) generalizes the relation, obtained in [15] for $s = \frac{1}{2}$, for the case of any spin values. For $n = 0$ the spectrum (5.3) is reduced to the one for a particle of any spin in Coulomb field (see (4.7)). We see that an arbitrary-spin particle as well as a spin-$\frac{1}{2}$ one has the coupled states in the considered combined field.

So we have obtained exact solutions of relativistic wave equations, describing the motion of any spin particle in some central external fields. Such solutions may be useful, e.q. in investigations connected with scarches of coupled states of exited atomic nucleus in magnetic-pole field and so on.

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