On the CP-noninvariant equations for the particle with zero mass and spin $s = \frac{1}{2}$

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One of us [1] has shown that for the particle with zero mass and spin $s = \frac{1}{2}$ there are three types of two-component equations (or one four-component equation with three different subsidiary conditions) which differ from one another by P, T and C properties. One of these equations is the two-component Weyl equation which, as is well known, is equivalent to the four-component Dirac equation

$$\gamma_\mu p^\mu \Psi(t, \boldsymbol{x}) = 0, \qquad \mu = 0, 1, 2, 3, \tag{1}$$

with the subsidiary relativistic invariant condition

$$(1 + \gamma_5)\Psi(t, \boldsymbol{x}) = 0. \tag{2}$$

Equations (1), (2) may be written in the form of a single equation [2]

$$\{\gamma_\mu p^\mu + \varkappa_1(1 + \gamma_5)\}\,\Psi(t, \boldsymbol{x}) = 0, \tag{3}$$

where $\varkappa_1$ is an arbitrary constant (not connected with mass of the particle. Equation (3) (or eqs. (1) and (2)) is P and C noninvariant, but CP-invariant.

In this note we give two other relativistic invariant equations which differ from (3) (or from (1) with the subsidiary condition (2)).

These equations have the form

$$\left\{\gamma_\mu p_\mu + \varkappa_2\left(1 + \gamma_5\frac{H}{E}\right)\right\}\Psi(t, \boldsymbol{x}) = 0, \tag{4}$$

$$\left\{\gamma_\mu p^\mu + \varkappa_3\left(1 + \frac{H}{E}\right)\right\}\Psi(t, \boldsymbol{x}) = 0, \tag{5}$$

$$H = \gamma_0\gamma_k p_k, \qquad k = 1, 2, 3, \qquad E = \sqrt{p_1^2 + p_2^2 + p_3^2}, \tag{6}$$

$\varkappa_2$, $\varkappa_3$ are arbitrary constants.

Equation (4) is equivalent to eq. (1) with the subsidiary condition

$$\left(1 + \gamma_5\frac{H}{E}\right)\Psi(t, \boldsymbol{x}) = 0. \tag{7}$$

Equation (5) is equivalent to eq. (1) with the subsidiary condition

$$\left(1 + \frac{H}{E}\right)\Psi(t, \boldsymbol{x}) = 0. \tag{8}$$

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The relativistic invariance of eqs. (4) and (5) (or the invariance of the subsidiary conditions (7) and (8)) follows from the fact that the operators γ_5 and H/E are invariants of the Poincaré group (for the case of zero mass).

It easy to verify that eq. (4) (or eq. (1) with condition (7)) is CP and CPT noninvariant.

Equation (5) (or eq. (1) with condition (8)) is P and T invariant (in the sense of Wigner time reflection), but C-noninvariant.

Equation (4) coincides with the equation obtained earlier [1] (where the substitution $e_3 = p_3/|p_3| \to H/E$ should be made).

Thus, as distinguished from eq. (3) (eqs. (1) and (2)) there are two more eqs. (4) and (5) which are also relativistic invariant, but CP-noninvariant.

A more detailed analysis of eqs. (4) and (5) will be given in another paper.

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2. Tokuoka Z., *Progr. Theor. Phys.*, 1967, **37**, 603;
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