Sailaubay Zhumatov (Laboratory of Dynamical Systems of Institute of Mathematics of Ministry of Education and Science of RK, Almaty, Kazakhstan)

On an Exponential Stability of Indirect Control Systems' Program Manifold

The problem of construction of all the set of differential equations possessing by given integral manifold has been formulated and a method of solving this problem is given in the work [1]. Later on Erugin's method was developed for construction of stable system of differential equations and nonlinear automatic control systems under given program manifold in [2], [3]. The problem of finding of exponential stability's conditions of indirect control systems' program manifold is investigated with respect to the given vector-function.

Let us consider the material system, possessing by $(n-s)$ - dimensional integral manifold $\Omega(t) \equiv \omega(t,x) = 0$, where the motion of which described by equations

$$\dot{x} = f(t,x) - B\xi, \quad \dot{\xi} = \varphi(\sigma), \quad \sigma = P^T\omega - R\xi. \tag{1}$$

Here $B \in \mathbf{R}^{n\times r}, P \in \mathbf{R}^{s\times r}, R > 0 \in \mathbf{R}^{r\times r}$ are matrices, $x \in \mathbf{R}^n$ is vector of objects state,$f \in \mathbf{R}^n$ is vector-function, $\omega \in \mathbf{R}^s$ is vector ,$\xi \in \mathbf{R}^r$ is vector of control on deflection from given program, satisfying of local quadratic connection's conditions. Taking into account that $\Omega(t)$ is integral manifold for the system (1) we will have $\dot{\omega} = \dfrac{\partial\omega}{\partial t} + Hf(t,x,\omega) = F(t,x,\omega),\ H = \dfrac{\partial\omega}{\partial x},\ F(t,x,0 \equiv 0)$ is Erugin's s -vector-function. Let $F = -A\omega,\ -A \in \mathbf{R}^{s\times s}$ is Hurwitz matrix. Then differentiating the manifold $\Omega(t)$ with respect to time t in view of (1), we derive that

$$\dot{\omega} = -A\omega - HB\xi, \quad \dot{\xi} = \varphi(\sigma), \quad \sigma = P^T\omega - R\xi. \tag{3}$$

Theorem 1. *Let the nonlinearity $\varphi(\sigma)$ satisfies of local quadratic connection's conditions, exists positive defined function $V(\omega,\xi)$, which derivative $-\dot{V} = W(\omega,\xi)$ in view of (3) is negative defined and is valid $\|z(t)\| \leq N\|z(t_0)\|exp[-\alpha(t-t_0)]$ for any $\omega(t_0,x_0)$ and $N > 0,\ \alpha > 0,\ \|z\|^2 = \|\omega\|^2 + \|\xi\|^2$. Then program manifold $\Omega(t)$ is exponential stable with respect to vector-function ω.*

References

1. Erugin N.P. // PMM. — Moskow, 6(1952), 659-670,

2. Galiullin A.S., Mukhametzyanov I.A., Mukharlyamov R.G. // Vestnik RUDN. — Moskow, 1(1994) 5-21,

3. Zhumatov, S.S., Krementulo, V.V., Maigarin, B.J. Second Lyapunov's method into the problems of stability and motion's control. Almaty: Nauka. 1999.