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On decompositions of semigroups of matrices

For the study of properties of semigroups with zero effective is a notion of 0-decomposition. The use of different types of 0-decompositions for structure description of semigroups with zero can be seen for example in [1-4].

Let M' denote the set of all matrices of all dimensions over a ring K and let $M = M' \cup \{0\}$, where $0 \notin M'$. Define a multiplication $\circ$ on M by the rule: $A \circ B = A \cdot B$ if $A, B \in M'$ and $A \cdot B$ is defined ($\cdot$ is the usual multiplication of matrices) and $A \circ B = 0$, otherwise. With this multiplication, M is a semigroup with the zero 0 and it called (complete) semigroup of matrices over K. Let I be an ideal of M consisting of 0 and of zero matrices of an arbitrary type. The factor semigroup M/I is called the reduced semigroup of matrices over R.

Let N be a set of all natural numbers. For $n \in N$ let $M_{n\times *}$ denote the union of all sets $M_{n\times k}, k \in N$ which are sets of all the matrices over a ring K of the type $n \times k, k \in N$, with 0 adjoined. For all $n, k \in N, n \neq k$ we put

$$F_{n\times k} = M_{n\times k} \setminus (M_{n\times n} M_{n\times k}),\ \ G_n = \{l \in N | l \neq n, F_{n\times l} \neq \emptyset\},$$

$$H = \cup_{k\in G_n} F_{n\times k},\ \ V = M_{n\times *} \setminus H,\ \ S_A = \{A, 0\}(A \in H).$$

A semigroup S with zero 0 is a left sum of semigroups $S_\alpha, \alpha \in Y$ if $S_\alpha \neq \{0\}$ for all $\alpha \in Y$, $S = \cup_{\alpha \in Y} S_\alpha$ and $S_\alpha \cap S_\beta = \{0\}$ and $S_\alpha S_\beta \subseteq S_\alpha$ for all $\alpha, \beta \in Y, \alpha \neq \beta$.

`Theorem.` $\{M_{n\times *} | n \in N\}$ is the greatest decomposition of complete semigroup of matrices M into a left sum of semigroups. If $G_n = \emptyset$, then semigroup $M_{n\times *}$ is indecomposable into a left sum of semigroups. If $G_n \neq \emptyset$, then $\{V, S_A | A \in H\}$ is the greatest decomposition of $M_{n\times *}$ into a left sum of semigroups.

Greatest decompositions of complete semigroup of matrices over a ring into a right and matrix sum of semigroups are described analogously. It specifies description of respective decompositions of M from [3]. Besides in according to problem 2 (see [3]) greatest decompositions of reduced semigroup of matrices M/I over a ring K into an orthogonal, left, right and matrix sum of semigroups are studied also.

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