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On the Generalized Topological Invariant of Pseudoharmonic Functions

Let $D \subset R^2$ be a closed domain bounded by a finite number of the closed Jordan curves $\gamma_0, \gamma_1, \ldots, \gamma_n$. We fix an orientation on D (i.e., for any γ_i there is an orientation that is consistent with other $\gamma_0^+, \gamma_1^-, \ldots, \gamma_n^-$).

Let $f : D \to R$ be a continuous function which satisfies the following conditions:

- $f\ |_{\gamma_i}$ is a continuous function with a finite number of local extrema for any $i \in \{0, \ldots, n\}$;
- $f\ |_{\mathrm{Int}D}$ has a finite number of critical points and each of them is a saddle point (in the neighborhood of it f has a representation like $Rez^n + const$, $z = x + iy$ and $n \geq 2$), where $\mathrm{Int}D = D\backslash_{(\bigcup\limits_i \gamma_i)}$.

Class of such functions coincides with class of *pseudoharmonic functions* [1].

We remind that f and g are called to be topologically equivalent if there exist the preserving orientation homeomorphisms $h_1 : D \to D$ and $h_2 : R \to R$ such that $f = h_2^{-1} \circ g \circ h_1$.

Let $f, g : D \to R$ be pseudoharmonic functions. What is the criteria for them to be topologically equivalent?

In [3] the problem was solved for the case n=0. We will be interested in case $n \geq 1$.

At first we construct the Reeb graphs of $f\ |_{\gamma_j}$, denote them by G_j, $j \in \{0, \ldots, n\}$, respectively, and put the orientation on G_0. Next, let us add to $\bigcup\limits_i G_i$ the collections of connected components $\widehat{f}^{-1}(a_i) \subseteq f^{-1}(a_i)$ and $\widehat{f}^{-1}(c_i) \subseteq f^{-1}(c_i)$ which contain only critical and boundary critical points, where every a_i is a critical value and every c_j is a semiregular value. Finally, we put a strict partial order on vertices by using the values of f. Denote by $\Delta(f)$ such structure.

We remark that $\Delta(f)$ is a finite connected oriented graph with a strict partial order and partially oriented.

Theorem Two pseudoharmonic functions f and g are topologically equivalent iff there is an isomorphism $\varphi : \Delta(f) \to \Delta(g)$ between their diagrams which preserves the strict partial order and the orientation given on them.

[1] Jenkins J.A, Morse M., *Contour equivalent pseudoharmonic functions and pseudoconjugates*, Amer.J.Math. **74** (1952), 23-51.

[2] Morse M., *The topology of pseudo-harmonic functions*, Duke Math.J.**13**(1946), 21–42.

[3] Yurchuk I., *Topological equivalence of fuctions from $F(D^2)$ class* , Zb. nauk. prac. Inst. Math. Ukr. **3**(3) (2006), 474–486 (in Ukrainian)