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An economical discretization strategy for ν-method

We consider the operator equation of the first kind

$$Ax = f, \tag{1}$$

in the Hilbert space X with a compact linear operator $A : X \to X$, $\mathrm{Range(A)} \neq \overline{\mathrm{Range(A)}}$, $f \in \mathrm{Range(A)}$. Suppose that instead of the exact right-hand side (1) we are given only a perturbation $f_\delta : \|f - f_\delta\| \leq \delta$, where $\delta > 0$ is a known error bound.

We shall construct approximations to the minimal norm solution $x^\dagger$ of (1) that satisfies the Holder-type source condition, i.e. $x^\dagger = |A|^\mu v$, $\|v\| \leq \rho$, $|A| = (A^*A)^{1/2}$, $\rho \geq 1$, $0 \leq \mu \leq 1$. The parameter μ is supposed to be unknown.

To stable approximation to the $x^\dagger$ it is required to apply regularization. For this purpose we apply the ν-method which for fixed $\nu = 1$ is a procedure of the following type: $x_0^\delta = 0$, $x_k^\delta = x_{k-1}^\delta + \sigma_k(x_{k-1}^\delta - x_{k-2}^\delta) + \omega_k A^*(y^\delta - Ax_{k-1}^\delta)$, $k = 1, 2, \ldots$, with σ_k, $\omega_k \geq 0$ depend on k.

To solve (1) we will consider projection methods using Galerkin information as discrete information about (1), i.e. scalar products (Ae_j, e_i), (f_δ, e_i), where $\{e_i\}_{i=1}^\infty$ is orthonormal basis in X, and indexes (i, j) are chosen from some bounded domain Ω of coordinate plane. The volume of Galerkin information used to approximate solve (1) characterizes economical properties of corresponding projection method.

To construct economical algorithm we use an adaptive approach to discretization that apply hyperbolic cross as domain Ω, and the discretization level is not fixed beforehand but selected during computations.

Proposed in the paper algorithm of solving (1) consists of combination such adaptive discretization strategy and the 1-method of regularization.

It is shown that for class of finitely smoothing operators the algorithm achieves the optimal order of accuracy recovering ($O(\delta^{\mu/(\mu+1)})$) of the solutions $x^\dagger$ and is more economical in the sense of amount of Galerkin information compare with methods from [2], where the problem of constructing economical projection methods for solving (1) was studied in the framework of traditional Galerkin discretization scheme using domain $\Omega = [1, m] \times [1, n]$.

[1] *Maas P., Pereverzev S.V., Ramlau R., Solodky S.G.* An adaptive discretization for Tikhonov-Phillips regularization with a posteriori parameter selection // Numer. Math. — 2001. — V. 87., no 1. — P. 485-502.

[2] *Plato R., Vainikko G.* On the Regularization of Projection Methods for solving Ill-posed Problems // Numer. Math. — 1990. — 57. — P. 63-79.