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On infinite-rank singular perturbations of the Schrödinger operator

Let $-\Delta$, $\mathcal{D}(\Delta) = W_2^2(\mathbb{R}^3)$ be the Schrödinger operator in $L_2(\mathbb{R}^3)$ and let $\mathfrak{U} = \{U_t\}_{t\in(0,\infty)}$ be the collection of unitary operators $U_t f(x) = t^{3/2} f(tx))$ in $L_2(\mathbb{R}^3)$ (so-called scaling transformations).

The operator $-\Delta$ is t^{-2}-homogeneous with respect to $\mathfrak{U}$ in the sense that

$$U_t \Delta u = t^{-2} \Delta U_t u, \quad \forall t > 0, \quad u \in W_2^2(\mathbb{R}^3).$$

In other words, the set $\mathfrak{U}$ determines the structure of a symmetry and the property of $-\Delta$ to be t^{-2}-homogeneous with respect to $\mathfrak{U}$ means that $-\Delta$ possesses a symmetry with respect to $\mathfrak{U}$.

Consider the heuristic expression

$$-\Delta + \sum_{i,j=1}^{\infty} b_{ij} < \psi_j, \cdot > \psi_i, \quad \psi_j \in W_2^{-2}(\mathbb{R}^3), \quad b_{ij} = \overline{b_{ji}} \in \mathbb{C}. \tag{1}$$

We will say that $\psi \in W_2^{-2}(\mathbb{R}^3)$ is $\xi(t)$-*invariant with respect to* $\mathfrak{U}$ if there exists a real function $\xi(t)$ such that

$$\mathbb{U}_t \psi = \xi(t) \psi, \quad \forall t > 0,$$

where $\mathbb{U}_t$ is the continuation of U_t onto $W_2^{-2}(\mathbb{R}^3)$.

Our aim is to study self-adjoint operator realizations of (1) assuming that all ψ_j are $\xi_j(t)$-invariant with respect to the set of scaling transformations $\mathfrak{U}$. In this way we generalize results of [1] to the case of infinite rank perturbations of the Schrödinger operator in $L_2(\mathbb{R}^3)$. In particular, the description of all t^{-2}-homogeneous extensions of the symmetric operator $-\Delta_{\rm sym}$ is obtained. Another interesting property obtained here is the possibility to get the Friedrichs and the Krein-von Neumann extension of $-\Delta_{\rm sym}$ as solutions of a system of equations involving the functions t^{-2} and $\xi(t)$.

[1] Hassi S., Kuzhel S. // J. Funct. Anal. — 2009. — **256**, 777-809.