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## On quivers of exponent matrices

Let  $M_n(Z)$  be a ring of square  $n \times n$ -matrices over the ring of integers.

**Definition.** A matrix  $\mathcal{E} = (\alpha_{ij}) \in M_n(Z)$  is called an exponent matrix if the following conditions hold:

- (i)  $\alpha_{ii} = 0$  for all  $i = 1, 2, \dots, n$ ;
- (ii)  $\alpha_{ik} + \alpha_{kj} \geq \alpha_{ij}$  for all  $i, j, k = 1, 2, \dots, n$ .

An exponent matrix  $\mathcal{E} = (\alpha_{ij})$  is called *reduced* if  $\alpha_{ij} + \alpha_{ji} > 0$  for all  $i \neq j$ .

Each exponent matrix can be related with some graph called a quiver. Let  $\mathcal{E}$  be a reduced exponent matrix,  $E$  be the identity one. Denote

$$\mathcal{E}^{(1)} = \mathcal{E} + E = (\beta_{ij}),$$
$$\mathcal{E}^{(2)} = (\gamma_{ij}), \quad \text{where } \gamma_{ij} = \min_k \{\beta_{ik} + \beta_{kj} - \beta_{ij}\}.$$

A graph  $Q(\mathcal{E})$  is called a *quiver* of an exponent matrix  $\mathcal{E}$  if

$$[Q(\mathcal{E})] = \mathcal{E}^{(2)} - \mathcal{E}^{(1)},$$

where  $[Q(\mathcal{E})]$  is the adjacency matrix of  $Q(\mathcal{E})$ .

The quiver of a reduced exponent matrix is a strongly connected simple laced graph but not each such graph is a quiver of some reduced exponent matrix.

We research quivers of reduced exponent matrices without loops and quivers with adjacency matrices that are multiple of stochastic matrices. We obtain lists of all such quivers with two, three and four vertices [1].

**Theorem.** *There exist quivers of reduced exponent matrices exactly:*

1. *one, two and eleven quivers without loops with two, three and four vertices respectively;*
2. *two, three and seventeen quivers with adjacency matrices that are multiple of stochastic matrices with two, three and four vertices respectively.*

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[1] Цюпій Т.І. Напівмаксимальні кільця та «х сагайдаки» // Вісник Київського університету. Серія: фізико-математичні науки. – 1999. – в.1.