Ukrainian Mathematical Congress – 2009

Tatiana Tchemisova, Alexander Plakhov (University of Aveiro, Aveiro,Portugal)

On numerical simulation in study aerodynamics and reverse Magnus effect in free molecular flow

We are concerned with a spinning solid body moving in a homogeneous medium. The medium is extremely rarefied, so the free path length of the medium particles is much larger than the body's size. In such a case, the interaction of the body with the medium can be described in terms of free molecular flow, where a flow of point particles falls on the body's surface; each particle interacts with the body and does not with other particles. We suppose that the medium particles stay at rest, that is, the absolute temperature of the medium equals zero. If consider the reference system moving forward together with the body, we obtain a parallel flow of particles falling on the body.

We neglect the angular momentum of particles; each particle is identified with a mass point that approaches the body, makes several (maybe none) reflections from its surface, and then goes away. All reflections are supposed to be *absolutely elastic.*

We restrict ourselves to the two-dimensional case. Consider a body contained in a circle of radius r and containing the concentric circle of radius $r - \varepsilon$ with $\varepsilon \ll r$. One can imagine a circle of radius r slightly damaged near the boundary, resulting in a *rough circle.* The shape of the body cannot change in time: it can only be translated or rotated. Denote by $\varphi(t)$ the rotation angle at the moment t, by $\omega(t)$ the angular velocity of the body, $\omega(t) = d\varphi/dt$, and let $\vec{v}(t)$ be the velocity of the body's center of mass. Let us agree to count off the rotation angle and the angular velocity clockwise.

Here we consider two problems as follows: (i) determine the force of the medium resistance acting on the body, find the moment of this force with respect to the body's center of mass, and investigate their dependence on the "shape of the roughness"; (ii) analyze the motion of the body in the medium, that is, study the functions $\varphi(t)$ and $\vec{v}(t)$.

Problem (i) is primary with respect to problem (ii). In the paper, we devote the main attention to problem (i), having just touched upon problem (ii); we will restrict ourselves with deducing equations of motion and solving these equations for some simple particular cases.

We present solutions of these problems obtained by numerical simulation using Xpress package. The main attention was focused on the problem, which is solved for the values $\lambda = 0, 0.1$, and 0.3. The method of solution is as follows: for N vectors $\vec{e}_i$ distributed in S^1 and equidistant, find the solution of the Monge-Kantorovich problem $\inf\langle \vec{R}(\nu, \lambda),\, \vec{e}_i\rangle =: r_i$. Here $\langle \cdot\,, \cdot\rangle$ stands for the scalar product. Then the convex polygon: the intersection of the half-planes $\langle \vec{r},\, \vec{e}_i\rangle \geq r_i$, is built. This polygon approximates the required set, and the approximation accuracy increases as n increases. The value $n = 100$ was used in our calculations.

There exist a number of papers that state that in the cases of movement in extremely rarefied media the reverse Magnus effect takes place, and study this phenomenon. In these studies the medium is supposed so rarefied that the description in terms of free molecular flow is possible.

In our opinion, the reverse Magnus effect in such kind of media is caused by two factors:

(i) Non-elastic collisions of particles with the body. A part of the tangential component of the momentum of particles is transmitted to the body, resulting in creation of a transversal force.

(ii) Multiple collisions of particles with the body originating from the fact that that the body's surface is not convex but contains microscopic cavities.

[1] K.I. Borg and L.,H. Sderholm. Orbital effects of the Magnus force on a spinning spherical satellite in a rarefied atmosphere. – Eur. J. Mech. B/Fluids, 27, 623-631 (2008).

[2] S. G. Ivanov and A.M. Yanshin. Forces and moments acting on bodies rotating around a symmetry axis in a free molecular flow. – Fluid Dyn., 15, 449 (1980).

[3] A. Plakhov. Billiard scattering on rough sets: Two-dimensional case. – SIAM J. Math. Anal., 40, 2155-2178 (2009). DOI. 10.1137/070709700.

[4] S. I. Rubinov and J. B. Keller. The transverse force on a spinning sphere moving in a viscous fluid. – J. Fluid Mech., 11, 447-459 (1961).

[5] P. D. Weidman and A. Herczynski. On the inverse Magnus effect in free molecular flow. – Physics of Fluids, 16, L9-L12 (2004).