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On invariant factors of block-triangular matrix and its diagonal blocks

It is well known that the matrix equation $AX - YB = C$ over commutative ring has a solution if and only if the matrices $\left\| \begin{matrix} A & 0 \\ C & B \end{matrix} \right\|$, $\left\| \begin{matrix} A & 0 \\ 0 & B \end{matrix} \right\|$ are equivalent. Over commutative elementary divisor ring R this condition is equivalent to the equality (up to associate) of the corresponding invariant factors of this matrices. Relation between invariant factors of the matrices $\left\| \begin{matrix} A & 0 \\ C & B \end{matrix} \right\|$, A, B are investigated in this paper.

Theorem. *Let D be a matrix over a commutative elementary divisor ring without zero divisors of the form*

$$D = \left\| \begin{matrix} D_1 & 0 \\ D_2 & D_3 \end{matrix} \right\| \sim diag(\overbrace{1, \ldots, 1, \underbrace{\varphi, \ldots, \varphi}_{s}}^{p+q}), \varphi \neq 0,$$

where D_1, D_3 be a square matrices with canonical diagonal forms

$$\Delta_1 = diag(\alpha_1, \alpha_2, \ldots, \alpha_q), \Delta_3 = diag(\beta_1, \beta_2, \ldots, \beta_p),$$

respectively.

I) If $p \leq s \leq q$, then α_i, β_j may be choosen so that

$$\alpha_1 = \alpha_2 = \ldots = \alpha_{q-s} = 1;$$

$$\alpha_{q-s+1}\beta_p = \alpha_{q-s+2}\beta_{p-1} = \cdots = \alpha_{q-s+p}\beta_1 = \varphi;$$

$$\alpha_{q-s+p+1} = \alpha_{q-s+p+2} = \ldots = \alpha_q = \varphi.$$

II) If $p, q \geq s$, then α_i, β_j may be choosen so that

$$\alpha_1 = \alpha_2 = \ldots = \alpha_{q-s} = 1;$$

$$\beta_1 = \beta_2 = \ldots = \beta_{p-s} = 1;$$

$$\alpha_{q-s+1}\beta_p = \alpha_{q-s+2}\beta_{p-1} = \cdots = \alpha_q\beta_{p-s+1} = \varphi.$$

III) If $p, q \leq s$, then α_i , β_j may be choosen so that

$$\alpha_1\beta_{q+p-s} = \alpha_2\beta_{q+p-s-1} = \cdots = \alpha_{q+p-s}\beta_1 = \varphi;$$

$$\alpha_{q+p-s+1} = \alpha_{q+p-s+2} = \cdots = \alpha_q = \beta_{q+p-s+1} = \beta_{q+p-s+2} \cdots = \beta_p = \varphi.$$

The obtained results can be applied in the case where R is a principal ideal domain in order to finding relations between the invariant factors of nonsingular block-triangular matrix

$$D = \left\| \begin{array}{cc} D_1 & 0 \\ D_2 & D_3 \end{array} \right\| \sim \Phi = diag(\varphi_1\ ,\ \ldots, \varphi_n)$$

with diagonal blocks, in the case $\frac{\varphi_2}{\varphi_1},\ \frac{\varphi_3}{\varphi_2},\ \ \ldots,\ \frac{\varphi_n}{\varphi_{n-1}}$ are pairwise relatively prime. Let $R_{(p)}$ denote the localization of R with respect to prime ideal (p). That is, $R_{(p)}$ is the ring consisting of entries of the form $x = p^\nu \frac{a}{b}$, where $a,\ b$ are in R and are relatively prime to p, $\nu \in N \cup \{0\}$. Than, in order that to find canonical diagonal forms of the matrices D_1 and D_3 we should find canonical diagonal forms of the matrices D_1 and D_3 in all localizations of the ring by irreducible (prime) divisors of element φ_n and to multiply the corresponding invariant factors of this matrices.
