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On J-self-adjoint extensions of a symmetric operator with zero characteristic function.

Let J be an involution (i.e., $J = J^*$, $J^2 = I$) in a Hilbert space $\mathfrak{H}$. We study J-self-adjoint extensions of a symmetric operator A_{sym} which is defined as follows: let U be a bilateral shift with a finite-dimensional wandering subspace W_0 in $\mathfrak{H}$ and let V be its restriction onto $\mathfrak{H} \ominus W_0$. Then

$$A_{\mathsf{sym}} = i(V+I)(V-I)^{-1}, \qquad \mathcal{D}(A_{\mathrm{sym}}) = \mathcal{R}(V-I). \tag{1}$$

In other words, A_{sym} is the restriction of the Cayley transformation of U:

$$A = i(U+I)(U-I)^{-1}, \qquad \mathcal{D}(A) = \mathcal{R}(U-I)$$

onto $\mathcal{D}(A_{\mathrm{sym}}) = \mathcal{R}(V-I)$. The operator A_{sym} is symmetric (since A is self-adjoint) and its deficiency induces coincide with $\dim W_0$.

The operator A_{sym} was constructed by Phillips [1] as an example of the symmetric operator which is invariant with respect to some set $\mathfrak{U}$ of unitary operators ($\mathfrak{U}$-invariant) but it has no $\mathfrak{U}$-invariant self-adjoint extensions. Later A.N. Kochubei showed that the characteristic function of A_{sym} is identically equal to zero [2].

Theorem 1 *Let A be a J-self-adjoint extension of A_{sym} and* $\dim W_0 = 2$. *Then either A has a real spectrum or the spectrum $\sigma(A)$ covers the whole complex plane $\mathbb{C}$ and its non-real part $\mathbb{C} \setminus \mathbb{R}$ consists of eigenvalues of A.*

If a J-self-adjoint extension A has a real spectrum then A is similar to a self-adjoint operator.

[1] R. S. Phillips // Proceedings of the International Symposium on Linear Spaces — Jerusalem, 1960. —366-398.

[2] A. N. Kochubei //Funk. Anal. Prilozh. —1979. —**13**, 77-78.