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Vortices of solutions of semi-stiff boundary value problems for the Ginzburg-Landau equation

We study the minimizers of the full Ginzburg-Landau free energy functional in the class $(u, A) \in H^1(G; \mathbb{C}) \times H^1(\mathbb{R}^2; \mathbb{R}^2)$ with $|u| = 1$ on ∂G, where G is a bounded domain in $\mathbb{R}^2$. Critical points (u, A) of the functional in this class are solutions of a semi-stiff boundary value problem for the Ginzburg-Landau equation, when the modulus of u satisfies the Dirichlet condition on ∂G while its phase satisfies the Neumann condition.

We consider two cases, either $G = \Omega$ or $G = \Omega \setminus \overline{\omega}$, where Ω and ω are bounded simply connected domains and $\overline{\omega} \subset \Omega$.

When $G = \Omega$, we consider the connected components of the class defined by the prescribed topological degree d of u on the boundary $\partial\Omega$. We show that for $d \neq 0$ the minimizers exist if $0 < \lambda \leq 1$ and do not exist if $\lambda > 1$, where $\sqrt{\lambda/2}$ is the Ginzburg-Landau parameter. We also establish the asymptotic locations of vortices for $\lambda \to 1 - 0$ (the critical value $\lambda = 1$ is known as the Bogomol'nyi integrable case).

In the case when G is doubly connected ($G = \Omega \setminus \omega$), we study the problem of minimization of the Ginzburg-Landau energy in the simplest nontrivial case when prescribing topological degree one on the outer component of the boundary and zero on the inner component of the boundary. We show that the minimizers exist if $0 < \lambda < 1$ and do not exist if $\lambda \geq 1$. In contrast to the case of simply connected domain vortices of minimizers converge to the boundary in the limit $\lambda \to 1 - 0$. The limiting positions of vortices are identified by the minimization problem $|\nabla V| \to \min$ on $\partial\Omega$, where V is the unique solution of the problem $\Delta V = V$ in G, $V = 1$ on $\partial\omega$ and $V = 0$ on $\partial\Omega$.