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Connection between Darlington representations of matrix functions of the class $\mathcal{C}^{p\times p}\Pi$ and their $J_{p,r}$-inner SI-dilations

The problem related to the Caratheodory class $\mathcal{C}^{p\times p}$ of analytical in open unit disk $D=\{z:|z|<1\}$ matrix functions $c(z)$ of size p with $\Re c(z)\geq 0$ in D is considered.

The necessery and sufficient condition when $c(z)$ admits Darlington representations

$$c(z)=T_{A(z)}[\tau]=[a_{11}(z)\tau+a_{12}(z)]\,[a_{21}(z)\tau+a_{22}(z)]^{-1}, \tag{1}$$

where τ is a constant matrix of size p with $\Re\tau\geq 0$ and $A(z)=[a_{ij}(z)]_{i,j=1,2}$ with blocks of size p is J_p-inner in D with signature matrix $J_p=\begin{bmatrix}0 & -I_p\\ -I_p & 0\end{bmatrix}$, was presented in [1]. It is: $c\in\mathcal{C}^{p\times p}\Pi$, i.e. $c\in\mathcal{C}^{p\times p}$ and it admits meromorphic pseudocontinuation in the exterior of D. In (1) matrix function $A(z)$ always can be chosen so that $\tau=\begin{bmatrix}I_r & 0\\ 0 & 0\end{bmatrix}$, where $r=rank\Re c(\zeta)$ a.e. on the unit circle $|\zeta|=1$.

From the other side representations of $c\in\mathcal{C}^{p\times p}\Pi$ as a diagonal 22-block of $J_{p,r}$-inner in D matrix function $\theta(z)$

$$\theta(z)=\begin{bmatrix}\alpha(z) & \beta(z) & 0\\ \gamma(z) & c(z) & I_p\\ 0 & I_p & 0\end{bmatrix}\quad\text{with}\quad J_{p,r}=\begin{bmatrix}I_r & 0 & 0\\ 0 & 0 & -I_p\\ 0 & -I_p & 0\end{bmatrix} \tag{2}$$

were considered in [2] and $\theta(z)$ were called $J_{p,r}$-inner SI-dilations of $c(z)$.

We study the bi-sided connection between representations (1) of the matrix functions $c\in\mathcal{C}^{p\times p}\Pi$ and their $J_{p,r}$-inner SI-dilations. We present three theorems: in theorem 1 this connection is established for the case $r=p$, in theorems 2 and 3 it is considered for the case $0<r<p$.

[1] Arov, D.Z. On Darlington method in research of dissipative systems (Russian) // Dokl. AN USSR. – 1971. – V. 201, 3. – P. 559–562.

[2] Arov, D.Z., Rozhenko, N.A. $J_{p,m}$-inner dilations of matrix-valued functions that belong to the Caratheodory class and admit pseudocontinuation // St. Petersburg Mathematical Journal. – 2008. – 19:3. – P. 375395.