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Tropical analysis of plurisubharmonic singularities

A semiring $S \subset \overline{\mathbf{R}}$ with an "addition" $a \oplus b = \max\{a, b\}$ (or $\min\{a, b\}$) and "multiplication" $a \otimes b = a + b$ is called tropical. For basics on tropical structures, see, e.g., [1] and the bibliography therein.

In the talk, we consider certain tropical semirings arising naturally in multidimensional complex analysis. This starts with a simple observation that a basic object of pluripotential theory – plurisubharmonic functions – can be viewed as Maslov's dequantization of analytic functions. To detect a tropical structure, we consider asymptotic behavior of absolute values $|f(z)|$ of analytic functions f when z approaches the zero set of f, which invokes investigation of singularities of plurisubharmonic functions and corresponding tropical semirings.

Standard characteristics of plurisubharmonic singularities are tropicalizations of classical notions from commutative algebra and can be viewed as functionals on the corresponding tropical semiring. Central role here is played by tropically linear functionals (i.e., additive and multiplicative with respect to the tropical operations, and homogeneous with respect to the usual multiplication by positive constants), a basic example being the Lelong number. Such functionals are tropicalizations of valuations on rings of germs of analytic functions.

A problem of description for linear functionals on plurisubharmonic singularities is posed. A representation is obtained for a larger class of the functionals, just tropically additive and positive homogeneous, as relative types with respect to maximal plurisubharmonic weights [2]. So, the description problem for the linear functionals reduces to characterizing plurisubharmonic weights that provide linearity of the relative types. In addition, tropically additive functionals are shown to be upper envelopes (that is, tropical "integrals") of tropically linear ones.

More details can be found in [3].

[1] G.L. Litvinov, The Maslov dequantization, idempotent and tropical mathematics: a very brief introduction // Idempotent mathematics and mathematical physics. Contemp. Math. — 2005. — **377**, Amer. Math. Soc., Providence, RI, 1–17.

[2] A. Rashkovskii, Relative types and extremal problems for plurisubharmonic functions // Int. Math. Res. Not. — 2006. — **2006**, Article ID 76283, 26 p.

[3] A. Rashkovskii, Tropical analysis of plurisubharmonic singularities // Preprint at http://arxiv.org/abs/0709.1826