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p-adic operational calculus of Mikusinski type

We consider Operational Calculus of Mikusinski type [1,2] based on space $C(\mathbb{Z}_p)$ of continuous functions from the ring of p-adic integers $\mathbb{Z}_p$ to the field of p-adic numbers $\mathbb{Q}_p$ [3,4]. We use canonical embedding and corresponding convolution

$$\mathcal{F} : C(\mathbb{Z}_p) \ni f \to \tilde{f}(y) := \sum_{n=0}^{+\infty} f(n)y^n \in \mathbb{Q}_p[[y]], \quad f * g := \mathcal{F}^{-1}[(\mathcal{F}f) \cdot (\mathcal{F}g)].$$

Space $C(\mathbb{Z}_p)$ with pointwise addition and convolution multiplication becomes an algebra. Standard norm

$$\|f\| = \sup_{x \in \mathbb{Z}_p} |f(x)|_p$$

turns out to be a valuation, and $C(\mathbb{Z}_p)$ is complete with respect to it.

We introduce a fraction field C/C and call it p-adic Mikusinski field or field of p-adic hyperfuctions. Surprisingly, C/C is incomplete with respect to valuation continued from $C(\mathbb{Z}_p)$.

We choose certain hyperfunctions corresponding to operations of shift $L \sim 1/y$, difference operator $\Delta \sim 1/y - 1$, operator of indefinite summation $S = \Delta^{-1}$, differentiation and integration. p-Adic exponential function $a^x \in C(\mathbb{Z}_p)$, $a \in 1 + p\mathbb{Z}_p$, is a particular case of p-adic hyperfunction $E_a := L/(L - a)$, $a \in \mathbb{Q}_p$.

Mahler base in $C(\mathbb{Z}_p)$ is easily described using p-adic hyperfunctions:

$$\left\{ \binom{x}{n} \right\} = \left\{ \frac{x(x-1)...(x-n+1)}{n!} \right\} = S^n(1+S) \in C/C, \quad n = 0, 1, 2, \dots .$$

Using Mahler expansion we embed convolution algebra of contionuous functions $C(\mathbb{Z}_p)$ and convolution algebra of p-adic measures $C(\mathbb{Z}_p)'$ into one field with valuation. More singular distributions are described in the same way.

[1] Mikusinski J. Operational calculus. Pergamon, Oxford, 1981.

[2] Yosida K. Operational Calculus. A theory of hyperfunctions. Springer-Verlag, New-York, 1984.

[3] Vladimirov V.S., Volovich I.V., Zelenov E.I. P-adic Analysis and Mathematical Physics. World Scientific, Singapore, 1994.

[4] Schikhof W. Ultrametric calculus. An introduction to p-adic analysis. Cambridge University Press, London, 1984.