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Nonlocal effects in homogenization of $p(x)$–Laplacian in perforated domains

We study the asymptotic behaviour, as $\varepsilon \to 0$, of u^ε solutions to a nonlinear elliptic equation with nonstandard growth condition in domains containing a grid–type microstructure $\mathcal{F}^\varepsilon$ that is concentrated in an arbitrary small neighborhood of a given hypersurface Γ. Mainly:

$$\begin{cases} -\mathrm{div}(|\nabla u^\varepsilon|^{p_\varepsilon(x)-2}\,\nabla u^\varepsilon) + |u^\varepsilon|^{\sigma(x)-2}\,u^\varepsilon = g(x) \quad \text{in } \Omega^\varepsilon; \\ u^\varepsilon = A^\varepsilon \text{ on } \partial\mathcal{F}^\varepsilon; \quad u^\varepsilon = 0 \text{ on } \partial\Omega; \quad \int\limits_{\partial\mathcal{F}^\varepsilon} |\nabla u^\varepsilon|^{p_\varepsilon(x)-2}\,\frac{\partial u^\varepsilon}{\partial \vec{\nu}}\,ds = 0, \end{cases} \tag{1}$$

where $\varepsilon > 0$; $\Omega^\varepsilon = \Omega \setminus \overline{\mathcal{F}^\varepsilon}$ is a perforated domain in $\mathbb{R}^n$ $(n \geq 2)$ with Ω being a bounded Lipschitz domain and $\mathcal{F}^\varepsilon$ being an open connected subset in Ω like a net that is concentrated near a hypersurface $\Gamma \Subset \Omega$; A^ε is an unknown constant; the growth functions p_ε and σ satisfy some natural conditions and g is a given function. Equations of such a type are known as $p_\varepsilon(x)$–Laplacian equations with a nonstandard growth condition. Without any periodicity assumption for a large range of perforated domains and by mean of the variational homogenization technique [1], we find the global behavior of $\{u^\varepsilon, A^\varepsilon\}$ as $\varepsilon \to 0$ which is described by the following non–local problem

$$\begin{cases} -\mathrm{div}\left(|\nabla u|^{p_0(x)-2}\,\nabla u\right) + |u|^{\sigma(x)-2}\,u = g(x) \quad \text{in } \Omega \setminus \Gamma; \\ u = 0 \text{ on } \partial\Omega;\ [u]_\Gamma^\pm = 0,\ \left[|\nabla u|^{p_0(x)-2}\,\frac{\partial u}{\partial \nu}\right]_\Gamma^\pm = c'_u(x, u - A) \text{ and } \displaystyle\int_\Gamma c'_u(x, u - A)\,dS = 0, \end{cases} \tag{2}$$

where $\lim\limits_{\varepsilon \to 0} A^\varepsilon = A < +\infty$ is an unknown constant, $c(x, u)$ is a given function, c'_u denotes the partial derivative of the function c. The general result is illustrated with a periodic example.

[1] V. A. Marchenko, E. Ya. Khruslov, Boundary value problems with fine–grained boundary, *Mat. Sbornik* **65**:3(1964), 458–472.