Maria Polekha (Candidate of Physical and Mathematical Sciences, Lecturer of Volyn National Lesya Ukrainka University, Lutsk, Ukraine)

Estimation in an implicit linear multivariate measurement error model

Consider the model of observations $DZ \approx 0$, where $Z \in \mathbb{R}^{(n+p)\times p}$ is unknown matrix parameter, which has to be estimated and the data matrix $D \in \mathbb{R}^{m\times(n+p)}$ is observed with errors. The main assumption was that we observe two independent copies of the model: $D(k)Z \approx 0,\ k = 1,2$, where $D(k) \in \mathbb{R}^{m_k\times(n+p)}$ are observed, $D(k) = \bar{D}(k) + \tilde{D}(k)$, $\bar{D}(k)Z = 0,\ k = 1,2$. Here $\bar{D}(k)$ are unknown nonrandom matrices and $\tilde{D}(k)$ are error matrices. We assumed that $\dim \operatorname{Ker}\bar{D}(k) = p,\ k = 1,2$, and total error covariance structure of data matrix is known up to two scalar factors. There exists n_1, $1 \le n_1 \le n+p-1$, such that $\tilde{D}(k) = \begin{bmatrix}\tilde{D}_1(k) & \tilde{D}_2(k)\end{bmatrix}$, $\tilde{D}_1(k) \in \mathbb{R}^{m_k\times n_1}$ and $\mathbf{E}\,\tilde{D}_1^\top(k)\tilde{D}_2(k) = 0$, and $\mathbf{E}\,\tilde{D}_j^\top(k)\tilde{D}_j(k) = \lambda_j^0 W_j(k)$, $j = 1,2$, where $W_j(k)$ are known positive semidefinite matrices and λ_i^0, are unknown positive scalars. The estimator of $V_p := \operatorname{Ker}\bar{D}(1) = \operatorname{Ker}\bar{D}(2)$ for increasing m_1 and m_2 under fixed n and p is constructed.

Based on the method of corrected objective function, the estimators $\hat{\lambda} := (\hat{\lambda}_1, \hat{\lambda}_2)$ of the scalar factors are proposed. Sufficient conditions of the consistency of the estimators are given.

Using both clusters we constructed a single subspace: $D_c := \begin{bmatrix} D(1) \\ D(2) \end{bmatrix}$ and $W_{cj} := W_j(1) + W_j(2),\ j = 1,2$. Then $D_c = \bar{D}_c + \tilde{D}_c$ and $H := \mathbf{E}\,\tilde{D}_c^T\tilde{D}_c = \begin{bmatrix} \lambda_1^0 W_{c1} & 0 \\ 0 & \lambda_2^0 W_{c2} \end{bmatrix}$. Define the matrix $\hat{H} = D_c^T D_c - \begin{bmatrix} \hat{\lambda}_1 W_{c1} & 0 \\ 0 & \hat{\lambda}_2 W_{c2} \end{bmatrix}$. Let $V_p(\hat{H})$ denotes the subspace spanned by the first p eigenvectors of $\hat{H}$ corresponding to the smallest eigenvalues, then $\hat{V}_p = V_p(\hat{H}) = \operatorname{span}(\hat{z}_1, \hat{z}_2, \dots, \hat{z}_p)$. Sufficient conditions of the consistency of the estimator $\hat{V}_p$ are given.

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