Anatoliy Petravchuk (Kyiv Taras Shevchenko University, Kyiv, Ukraine)

On behavior of solvable ideals of Lie algebras under outer derivations

Let L be a finite dimensional Lie algebra over a field of characteristic 0. It is well-known that its solvable radical $S(L)$ is a characteristic ideal of L, i.e. $D(S(L)) \subseteq S(L)$ for every derivation $D \in Der(L)$. This result breaks down in characteristic $p > 0$ (see for example [1], p.74-75). The noted counter-example has solvable radical of derived length $[\log_2 p] + 1$. We prove that the solvable radical $S(L)$ is a characteristic ideal if its derived length is less that $\log_2 p$.

Derivations of Lie algebras and associative algebras were studied by many authors (see for example [2], [3]). In particular, the behavior of locally nilpotent ideals of Lie algebras (in characteristic 0) under derivations was studied in [2].

Theorem 1. Let L be a Lie algebra over a field F and let I be its solvable ideal of derived length n. Then the ideal $I + D(I)$ is solvable and its derived length $\leq 2n$ in the following cases: 1) $char F = 0$; 2) $n < \log_2 p$ where $p = char F > 0$.

The next Theorem is the main result of the paper. It follows immediately from Theorem 1.

Theorem 2. Let L be a Lie algebra over a field F and let $S(L)$ be the sum of all solvable ideals of L. Then $S(L)$ is a characteristic ideal of the algebra L in the following cases: 1) $char F = 0$; 2) $S(L)$ is solvable of derived length $< \log_2 p$ where $p = char F > 0$.

[1] N. Jacobson, *Lie algebras*, Interscience tracts , no.10 (New York, 1962).

[2] B. Hartley, *Locally nilpotent ideals of a Lie algebra*, Proc. Cambridge Phil. Soc., **63** (1967), 257–272.

[3] George B. Seligman, *Characteristic ideals and the structure of Lie algebras*, Proceedings of the AMS, vol. 8, no.1, (1957), 159–164.