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On existence of global V-bounded solutions for nonlinear systems of ODEs

The goal of this report is to lay down sufficient conditions under which the nonlinear nonautonomous system of ODEs

$$\dot{x} = f(t, x) \tag{1}$$

where $f : \Omega \mapsto \mathrm{R}^n$ ($\Omega \subset \mathrm{R}^{1+n}$) has a global solution $x(t)$ extendable on the entire time axis and possessing the property that a given auxiliary function $V(t, x)$ (a time dependent norm surrogate) is bounded along the graph of $x(t)$. We especially focus on getting estimates for the function $V(t, x(t))$. The main results are obtained by using the Ważewski topological principle.

To apply the the Ważewski principle, along with the function V we use another auxiliary function $W(t, x)$. In general case, this function is a sign-changing one, but it must have positive derivative by virtue of the system (1) in the domain where $V \geq v_0$ for some constant $v_0 > 0$. We call V and W *the estimating function* and *the guiding function* respectively and we say that together they form the V–W-pair of the system.

We prove two main theorems concerning the existence and the uniqueness of V-bounded solution to a nonlinear nonautonomous system possessing V–W-pair. We also show how the estimating and guiding functions are constructed by means of nonautonomous quadratic forms.

[1] Lagoda V., Parasyuk I. // arXiv:0901.0234 [math. CA]. — 2009.