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On realizability of permutation groups as isometry groups

We call a permutation group (G, X) *realizable* as isometry group if there exists a metric d_X on the set X such that (G, X) is isomorphic to the isometry group IsX of the space (X, d_X) as a permutation group.

We say that the operation $*$ on the class of permutation groups *preserves realizability* if for arbitrary realizable permutation groups (G, X) and (H, Y) the permutation group $(G, X) * (H, Y)$ is realizable as well.

Theorem 1 *Operations of direct sum and wreath product (see, for example, [1]) preserve realizability.*

Let $\{T_1, T_2, \dots, T_n, \dots\}$ be some alphabet. We define special formulas in this alphabet as follows:

- T_i is a *formula* $(i = 1, 2, \dots)$.
- If H_1 and H_2 are *formulas* then $(H_1 \wr H_2)$, $(H_1 \oplus H_2)$ are *formulas*.
- There are no other formulas.

For a formula $W(T_1, T_2, \dots, T_n)$ its value on a sequence (G_1, X_1), (G_2, X_2), …, (G_n, X_n) of permutation groups is the permutation group

$$W((G_1, X_1), (G_2, X_2), \dots, (G_n, X_n)).$$

Corollary 1 *Let $W(T_1, T_2, \dots, T_n)$ be a formula. For arbitrary realizable permutation groups (G_1, X_1), (G_2, X_2), …, (G_n, X_n) the permutation group*

$$W((G_1, X_1), (G_2, X_2), \dots, (G_n, X_n))$$

is realizable.

Theorem 2 *A regular group G of order $n \geq 3$ is realizable iff G is an elementary abelian 2-group.*

[1] Kaloujnine L.A., Beleckij P.M., Feinberg V.T. Kranzprodukte. — Leipzig: BSB B.G. Teubner Verlagsgesellschaft, 1987.