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On a differentially prime radicals and submodules

Let (R, Δ) be an associative differential ring with nonzero identity, and let (M, D) be a differential module over (R, Δ), where $\Delta = \{\delta_1, \dots, \delta_n\}$ is the set of pairwise commutative ring derivations, $D = \{d_1, \dots, d_n\}$ is the set of module derivations consistent with the corresponding ring derivations δ_i.

A differentially prime radical $r_d(N)$ ([1]) of the differential submodule N of M is the intersection of all the differentially prime submodules of M containing N.

Proposition. *For any differential submodules N of the differential modules M, the differentially prime radical $r_d(N)$ has the following properties*

1. $r_d(N)$ is a d-prime d-submodules of M;
2. $N \subseteq r_d(N) \subseteq M$;
3. $r_d(N) = r_d(r_d(N))$;
4. If $N_1 \subseteq N_2$, then $r_d(N_1) \subseteq r_d(N_2)$.
5. $r_d(N_1 \cdot N_2) = r_d(N_1 \bigcap N_2) = r_d(N_1) \bigcap r_d(N_2)$;
6. $r_d(N_1 + N_2) = r_d(r_d(N_1) + r_d(N_2))$.

Theorem 1. *The set of all differentially prime differential radicals forms a lattice, which is complete and distributive.*

Theorem 2. *The class of differentially prime differential modules is closed with respect to ultraproducts.*

A question on ultraclosedness of a class of Sdm-systems, quasi-prime differential submodules and multiplicalive differential modules is also considered.

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