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Schrödinger operators with pseudopotentials of the type $\alpha_1\delta'(x) + \alpha_2\delta'(x)$

We consider one center Hamiltonians of the form

$$H(\alpha_1, \alpha_2) = -\frac{d^2}{dx^2} + U(x) + \alpha_1\delta'(x) + \alpha_2\delta'(x) \tag{1}$$

here U is a smooth potential, α_1 and α_2 are coupling constants attached to the point source located at the origin, and $\delta'(x)$ is the derivative of the Dirac function. The quantum mechanical particle thus moves under the influence of the potential U perturbed by a contact potential created by point sources of strength α_1 and α_2 located at the origin. We approximate (1) by operators $\mathcal{H}_{\varepsilon,\gamma}(\alpha_1, \alpha_2, \Psi, \Phi)$ that are the closure of essentially selfadjoint ones

$$H_{\varepsilon,\gamma}(\alpha_1, \alpha_2, \Psi, \Phi) = -\frac{d^2}{dx^2} + U(x) + \frac{\alpha_1}{\varepsilon^2}\Psi(\varepsilon^{-1}x) + \frac{\alpha_2}{\varepsilon^{2\gamma}}\Phi(\varepsilon^{-\gamma}x)$$

with domain $C_0^\infty(\mathbf{R})$. Here Ψ and Φ are the shapes of the δ'-like barrier that is to say $\varepsilon^{-2}\Psi(\varepsilon^{-1}x)$ and $\varepsilon^{-2\gamma}\Psi(\varepsilon^{-\gamma}x)$ tend to $\delta'(x)$ in the sense of distributions, $\gamma \geq 1$.

Applying asymptotic analysis we can construct the limit selfadjoint operators $\mathcal{H} = \mathcal{H}_\gamma(\alpha_1, \alpha_2, \Psi, \Phi)$ corresponding to (1). The crucial point is that the asymptotic behavior of eigenvalues and eigensubspaces of $\mathcal{H}$ strongly depends on the spectra of the Sturm-Liouville problems with an indefinite weight functions Ψ and Φ. Methods of operator theory in the Krein spaces are used here.

[1] Albeverio S., Gesztesy F., Høegh-Krohn R., Holden H. Solvable models in quantum mechanics. With an appendix by Pavel Exner. — Chelsea Publishing, 2005.