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On asymptotic stability of neural network on the time scale

In this paper we consider stability of a *neural network on time scale* the dynamics of which is described by equation of the type

$$x^{\Delta}(t) = -Bx(t) + TS(x(t)) + J, \quad t \in [0, +\infty), \tag{1}$$

whose solution $x(t; t_0, x_0)$ for $t = t_0$ takes the value x_0, i.e.

$$x(t_0; t_0, x_0) = x_0, \quad t_0 \in [0, +\infty), \quad x_0 \in R^n,$$

where $t \in \mathbf{T}$, $\mathbf{T}$ is an arbitrary time scale [1], $0 \in \mathbf{T}$, $\sup \mathbf{T} = +\infty$. In system (1) $x^{\Delta}(t)$ is a Δ-derivative on time scale $\mathbf{T}$, вектор $x \in R^n$ characterizes the state of neurons, $T = \{t_{ij}\} \in R^{n \times n}$, the components t_{ij} describe the interaction between the i-th and j-th neurons, $S\colon R^n \to R^n$, $S(x) = (s_1(x_1), s_2(x_2), \ldots, s_n(x_n))^{\mathrm{T}}$, the function s_i describes response of the i-th neuron, $B \in R^{n \times n}$, $B = \mathrm{diag}\{b_i\}$, $b_i > 0$, $i = 1, 2, \ldots, n$, $J \in R^n$ is a constant external input vector.

We assume on system (1) as follows.

S_1. The vector-function $f(x) = -Bx + TS(x) + J$ is regressive.

S_2. There exist positive constants $L_i > 0$, $i = 1, 2, \ldots, n$, such that $|s_i(u) - s_i(v)| \le L_i|u - v|$ for all $u, v \in R$.

By second Liapunov method sufficient conditions of asymptotical stability for equilibrium of the neural system (1) on time scales are obtained. The sufficient conditions of regressivity of system's function $f(x) = -Bx + Ts(x) + J$ end existence conditions of unique equilibrium are given.

The efficiency of the obtained sufficient conditions on the numerical example is tested.

[1] M. Bohner, A.A. Martynyuk. // Nonlinear dynamics and systems theory. — 2007. — **7**, N 3.