Alexander Kosyak (Institute of Mathematics, Ukrainian National Academy of Sciences, Kiev, Ukraine)

Representations of the infinite-dimensional groups and the Ismagilov conjecture

Sir Michael Atiyah said in his Fields Lecture [1]: " ... the 21th century might be the era of quantum mathematics or, if you like of infinite-dimensional mathematics."

We start a systematic development of a *noncommutative harmonic analysis on infinite-dimensional groups*. These groups are supposed to be non-locally compact. Since almost all constructions in the harmonic analysis on a *locally compact group* G are based on the existence and uniqueness of the G-invariant measure (*Haar measure*) on the group G, it is very natural to try to construct something similar for non-locally compact groups. Since the initial group G is not locally compact there is neither Haar (invariant) measure (A. Weil, [3]), nor G-quasi-invariant measure (Xia Dao-Xing, [4]) on it. The most direct approach to construct an analog of the Haar measure is as follows (A.V. Kosyak, [2]).

Try to *construct some bigger topological group* $\tilde{G}$ containing the initial group G as a *dense subgroup* (that is $\tilde{G}$ *is a completion of* G) and G*-right-quasi-invariant measure* μ on $\tilde{G}$. Thus, the starting point is to *construct* for an infinite-dimensional group G a *triplet* $(\tilde{G}, G, \mu)$ with the mentioned properties. In such a way we construct *regular, quasiregular and induced representations* (depending on the completion $\tilde{G}$ and the measure μ) for the infinite-dimensional groups and study their properties.

Ismagilov's conjecture and its generalizations explain in terms of the corresponding measures, when these representations are *irreducible*. We study the *von Neumann algebra* $\mathfrak{A}^{R,\mu}(G) = (T_t^{R,\mu} | t \in G)''$, generated by the right $T^{R,\mu}$ (or left) regular representations of the infinite-dimensional nilpotent groups $B_0^{\mathbb{N}}$ and $B_0^{\mathbb{Z}}$, where M' is *commutant of the algebra* M. Firstly, we give a condition on the measure μ for the right von Neumann algebra $\mathfrak{A}^{R,\mu}(G)$ to be the *commutant* of the left one $\mathfrak{A}^{L,\mu}(G)$. This is an analog of the well-known *Dixmier commutant theorem* for locally compact groups. Secondly we study when the von Neumann algebra M generated by the right (or left) regular representations is *factor*, i.e. when $M \bigcap M'$ is trivial, i.e. consists of scalar operators. Finally we show that the corresponding *factors are of type* III_1 under some natural conditions on the measure μ.

[1] M. Atiah. Mathematics in the 20th Century, Authors Fields Lecture at the Word Mathematical Year 2000 Symposium, Toronto, June 7–9, 2000.

[2] A.V. Kosyak. Representations of the infinite-dimensional groups and the Ismagilov conjecture, 453 p. (in preparation).

[3] A. Weil. L'intégration dans les groupes topologiques et ses applications 2^e ed. — Paris: Hermann, 1953.

[4] Xia-Dao-Xing. Measures and Integration in Infinite-Dimensional Spaces. — New York/London: Academic Press, 1978.