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1-D Schrödinger operators with local point interactions

Spectral properties of 1-D Schrödinger operators

$$\mathrm{H}_{X,\alpha} := -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \sum_{x_n \in X} \alpha_n \delta(x - x_n)$$

with local point interactions on a discrete set $X = \{x_n\}_{n=-\infty}^{\infty} \subset \mathbb{R}$ $(\pm x_n \uparrow +\infty,\ n \to \pm\infty)$, are well studied when $d_* := \inf_{n\in\mathbb{Z}}(x_n - x_{n-1}) > 0$ (numerous results and a comprehensive list of references may be found in [1]; see also a survey of recent results given by Exner in [1, Appendix K]). In the case $d_* = 0$, it is only known that the operator $\mathrm{H}_{X,\alpha}$ may be symmetric with nontrivial deficiency indices (see [3]).

The main aim of our talk is the spectral analysis of the operators $\mathrm{H}_{X,\alpha}$ when $d_* = 0$. We show that the spectral properties of $\mathrm{H}_{X,\alpha}$ like self-adjointness, discreteness, and lower semiboundedness correlate with the corresponding spectral properties of certain classes of Jacobi matrices. Based on this connection, we obtain necessary and sufficient conditions for the operators $\mathrm{H}_{X,\alpha}$ to be self-adjoint, lower-semibounded, and discrete in the case $d_* = 0$.

[1] Albeverio S., Gesztesy F., Hoegh-Krohn R., Holden H. Solvable Models in Quantum Mechanics. —AMS Chelsea Publ., 2005.

[2] Albeverio S., Kurasov P. Singular Perturbations of Differential Operators and Schrödinger Type Operators. — Cambridge Univ. Press, 2000.

[3] Shubin Christ C., Stolz G. // J. Math. Anal. Appl. —1994. —**184**, 491–516.

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