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On Differential Preradical Filters

Let A be a differential ring and let I be a differential ideal of A.

Put

$$(I : a^{\infty}) = \{u | u \in A \& \forall n \in \{0, 1, 2 \ldots\} : ua^{(n)} \in I\}, (a \in A).$$

The set of all differential left ideals of A will be denoted by $LD(A)$.

A differential preradical filter of A is a collection F of differential left ideals of A possessing the following properties:

(1) $I \in F \& I \subseteq J \& J \in LD(A) \longrightarrow J \in F$;

(2) $I \in F \& a \in A \longrightarrow (I : a^{\infty}) \in F$;

(3) $I \in F \& J \in F \longrightarrow I \cap J \in F$.

A differential radical filter of A is a differential preradical filter F of A possessing the following property:

(4) $I \in LD(A) \& I \subseteq J \& J \in F \& (\forall a \in J : (I : a^{\infty}) \in F) \longrightarrow I \in F)$.

Proposition 1. If F is a collection of differential left ideals of A possessing (2), (4) then F possesses (3).

Proposition 2. Let S be a differential two-sided ideal of A. If every differential left ideal of A is two-sided then the set

$$F_S = \{T | T \in LD(A) \& S + T = A\}$$

is a differential radical filter of A .

Proposition 3. If I is a differential idempotent two-sided ideal of A then $s : M \longmapsto s(M)$ $(s(M) = \{m \in M | \forall a \in I \forall n \in \{0, 1, 2, ...\}) : m^{(n)}a = 0\}\}$ is a differential hereditary radical.