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The Upper Estimate on the Number of Limit Cycles of Even Degree Liénard Equations in the Focus Case

In 1998 S. Smale [4] suggested to consider a restriction of the second part of the Hilbert's 16th problem to a special class of polynomial vector fields on the plane. This class is called Liénard equations:

$$\begin{cases} \dot{x} = y - F(x) \\ \dot{y} = -x \end{cases} . \tag{1}$$

For the case of odd degree $n = \deg F(x)$ in 1999 Yu. Ilyashenko and A. Panov [2] got an explicit upper bound for the number of limit cycles of (1) through n and magnitudes of coefficients of $F(x)$. Their result reclined on the theorem of Ilyashenko and Yakovenko that binds the number of zeros and the growth of a holomorphic function [3].

In 2008 M. Caubergh and F. Dumortier give explicit upper estimates for large amplitude limit cycles of even degree ($n = 2l$) Liénard equations [1]. They proved that there exists such radius R, that there are at most l limit cycles having an intersection with the ball around the origin with the radius R.

We give an explicit upper bound for the number of limit cycles of Liénard equations of even degree in the case its unique singular point is a focus. Our estimate (a triple exponent on n) depends on four parameters: n, C, a_1, R.

Let $F(x) = x^n + \sum_{i=1}^{n-1} a_i x^i$, where F is the monic polynomial without constant term, such that $\forall i$: $|a_i| < C$, so C is the size of the compact subset in the space of parameters and $|a_1|$ stands the distance from the equation linearization to the center case in the space of parameters.

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