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Sharp inequalities of Nagy-Kolmogorov type in the spaces of Weil

For $r \in \mathbf{N}$ and $p \in (0, \infty]$ we denote by $L^r_{p,\infty}(\mathbf{R})$ the space of all functions $x : \mathbf{R} \to \mathbf{R}$ for which $x^{(r-1)}$ are locally absolutely continuous, $x \in L_p(\mathbf{R})$ and $x^{(r)} \in L_\infty(\mathbf{R})$.

We prove the following sharp inequality of Nagy-Kolmogorov type

$$\left\|x^{(k)}\right\|_{W_q} \le \frac{\|\varphi_{r-k}\|_{W_q}}{L(\varphi_r)_p^\alpha} L(x)_p^\alpha \left\|x^{(r)}\right\|_\infty^{1-\alpha}, \quad x \in L^r_{p,\infty}(\mathbf{R}),$$

in the cases 1) $k = 0$, $q \ge p$, 2) $1 \le k \le r-1$, $q \ge 1$, where φ_r is the perfect Euler's spline of order r, $\alpha = (r-k)/(r+1/p)$,

$$\|x\|_{W_q} := \lim_{\Delta\to\infty} \sup_{a\in\mathbf{R}} \left(\frac{1}{\Delta} \int\limits_a^{a+\Delta} |x\,(t)|^q \, dt \right)^{1/q},$$

$$L(x)_p := \sup \left\{ \left(\int\limits_a^b |x(t)|^p dt \right)^{\frac{1}{p}} : \ a, b \in \mathbf{R}, \ |x(t)| > 0, \ t \in (a,b) \right\}.$$

Besides, we show that for any ω, γ, $\delta > 0$ there exists a function $x \in L^r_{p,\infty}(\mathbf{R})$ such that

$$\left\|x^{(k)}\right\|_{W_q} = \omega, \quad L(x)_p = \gamma, \quad \left\|x^{(r)}\right\|_\infty = \delta$$

if and only if

$$\omega \le \frac{\|\varphi_{r-k}\|_{W_q}}{L(\varphi_r)_p^\alpha} \, \gamma^\alpha \, \delta^{1-\alpha}.$$

We established also the following generalization of Calderon and Klein's inequality for the entire functions f of exponential type σ:

$$\left\|f^{(k)}\right\|_{W_q} \le \sigma^k \left\{ \frac{1}{\pi} \int\limits_0^\pi |\cos t|^q dt \right\}^{1/q} \|f\|_\infty, \quad k \in \mathbf{N}, \quad q \ge 1.$$