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Radical functor in the concrete category

Usually radicals are studied in the categories of modules or rings. We shall generalize this notion for an arbitrary category.

Let $\mathcal{C}$ be an arbitrary concrete category. (Though all these things we can do in an arbitrary category.)

Definition. Let T_1 and T_2 be functors from the category $\mathcal{A}$ to the category $\mathcal{B}$. The functor T_1 is called a subfunctor of the functor T_2 (denote $T_1 \leqslant T_2$) if $T_1(A)$ is a subobject of $T_2(A)$ (denote $T_1(A) \subseteq T_2(A)$) for every $A \in Ob(\mathcal{A})$ and the following diagram

$$\begin{array}{ccc} T_1(A_1) & \xrightarrow{T_1(\varphi)} & T_1(A_2) \\ {\scriptstyle i_1}\big\downarrow & & {\scriptstyle i_2}\big\downarrow \\ T_2(A_1) & \xrightarrow{T_2(\varphi)} & T_2(A_2) \end{array}$$

is commutative for every morphism $\varphi\colon A_1 \to A_2$, $A_1, A_2 \in Ob(\mathcal{A})$.

Definition. The functor T_1 is called a normal subfunctor of the functor T_2 if $T_1(A)$ is a normal subobject of $T_2(A)$ for every $A \in Ob(A)$.

Definition. Let $\mathcal{A}$ be a category, T_1 and T_2 be functors on $\mathcal{A}$, such that T_1 is a normal subfunctor of T_2. A factor-functor T_2/T_1 is a functor such that $(T_2/T_1)(A) = T_2(A)/T_1(A)$ $\forall A \in Ob(\mathcal{A})$ and the next diagram is commutative

$$\begin{array}{ccc} T_1(A_1) & \xrightarrow{T_1(\varphi)} & T_1(A_2) \\ {\scriptstyle i_1}\big\downarrow & & {\scriptstyle i_2}\big\downarrow \\ T_2(A_1) & \xrightarrow{T_2(\varphi)} & T_2(A_2) \\ {\scriptstyle \pi_1}\big\downarrow & & {\scriptstyle \pi_2}\big\downarrow \\ T_2(A_1)/T_1(A_1) & \longrightarrow & T_2(A_2)/T_1(A_2), \end{array}$$

where i_1, i_2 are normal monomorphisms, π_1, π_2 are canonical epimorphisms.

Definition. A preradical functor T on the category $\mathcal{A}$ is called a radical functor if $T(I/T) = 0$, where I is an identity functor.

[1] Kilp M., Knauer U., Mikhalev A. V. Monoids, acts and categories: with applications to wreath products and graphs; a handbook for students and researchers. — Berlin; New York: de Gruyter, 2000.

[2] Mitchell B. Theory of categories. — New York: London Acad. Press, 1965.