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## Linear operators satisfying a polynomial relation on a space with positive semidefinite form

This is joint work with Vladimir V. Sergeichuk.

By a *quasi-unitary space*, we mean a finite dimensional complex vector space $V$ with scalar product given by a positive semidefinite Hermitian form. By a *quasi-orthogonal basis* of $V$, we mean a basis in which the matrix of scalar product has the form $I_m \oplus 0$; that is, the scalar product is $(u, v) = \bar{u}_1 v_1 + \cdots + \bar{u}_m v_m$. Correspondingly, we partition the matrix of each linear operator $\mathcal{A}$ in this basis into blocks:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}_m, \qquad A_{11} \text{ is } m\text{-by-}m.$$

The index $m$ of the matrix defines its partition into blocks. This index is equal to the rank of the scalar product, and so it defines the quasi-unitary space up to isomorphism.

In the following theorems, we classify operators satisfying a polynomial equation of degree 2. If the degree is $\geq 3$, then the classification problem is *unitarily wild*; i.e, it contains the problem of classifying *arbitrary* operators on unitary spaces.

**Theorem 1.** *Let $\mathcal{A}$ be a projector (i.e., $\mathcal{A}^2 = \mathcal{A}$) on a quasi-unitary space. Then there is a quasi-orthogonal basis, in which the matrix of $\mathcal{A}$ is a direct sum, uniquely determined up to permutation of summands, of block matrices of the form:*

$$[0]_m,\ [1]_m\ (m \in \{0,1\}), \quad \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}_1, \quad \begin{bmatrix} 1 & 0 \\ 1 & a \end{bmatrix}_2\ (a \in \mathbb{R},\ a > 0). \tag{1}$$

**Theorem 2.** *Let $\mathcal{A}$ be a selfannihilating operator (i.e., $\mathcal{A}^2 = 0$) on a quasi-unitary space. Then there is a quasi-orthogonal basis, in which the matrix of $\mathcal{A}$ is a direct sum, uniquely determined up to permutation of summands, of block matrices of the form:*

$$[0]_m, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}_m\ (m \in \{0,1\}), \quad \begin{bmatrix} 0 & 0 \\ a & 0 \end{bmatrix}_2, \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}_1\ (a \in \mathbb{R},\ a > 0). \tag{2}$$

**Theorem 3.** *Let $\mathcal{A}$ be an operator on a quasi-unitary space.*

(a) *If $f(x) = (x - \alpha)(x - \beta) \in \mathbb{C}[x]$, $\alpha \neq \beta$, and $f(\mathcal{A}) = 0$, then there is a quasi-orthogonal basis, in which the matrix of $\mathcal{A}$ is a direct sum, uniquely determined up to permutation of summands, of matrices $(\alpha - \beta)B + \beta I$, where $B$ is of the form* (1).

(b) *If $f(x) = (x - \alpha)^2 \in \mathbb{C}[x]$ and $f(\mathcal{A}) = 0$, then there is a quasi-orthogonal basis, in which the matrix of $\mathcal{A}$ is a direct sum, uniquely determined up to permutation of summands, of matrices $\alpha I + B$, where $B$ is of the form* (2).

(c) *If $f(x) \in \mathbb{C}[x]$ is of degree $\geq 3$, then the problem of classifying operators $\mathcal{A}$ on a quasi-unitary space satisfying $f(\mathcal{A}) = 0$ is unitarily wild.*