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Small deviation of Brownian motion with large drift

Let $B(t)$ be standard Brownian motion, and let $f(t)$, $g(t)$ are smooth functions with $f(0) = 0, g(t) > 0$, $t \geq 0$. We investigate of exact asymptote the following probability :

$$P_\lambda = \mathbf{P}\left(|B(t) + \lambda f(t)| \leq \lambda^{-\alpha} g(t),\, t \in [0,T]\right),\, \alpha > 0,\, \lambda \to \infty.$$

Various authors have studied this probability for one-sided boundary case. See [1, 2] and the references given there.

Theorem. *Let the following conditions hold*

1. $f \in C^2_{[0,T]}$, $f(0) = 0$.
2. $g \in C^1_{[0,T]}$, $\min\limits_{t\in[0,T]} g(t) > 0$.
3. $\alpha > 1, \qquad T > 0$.

Then, at $\lambda \to \infty$ the following presentation takes place

$$P_\lambda = 2\pi \frac{1 + exp(-\kappa)}{\kappa^2 + \pi^2} \times$$

$$\times \exp\left\{ -\frac{\lambda^2}{2} \int\limits_0^T \dot{f}^2(s)ds - \frac{\lambda^{2\alpha}\pi^2}{8} \int\limits_0^T g^{-2}(s)ds + \lambda^{1-\alpha}\left(g(0)\dot{f}(0) + \int\limits_0^T \dot{g}(s)\dot{f}(s)ds \right) \right\} \times$$

$$\times \left(1 + O(\lambda^{-2\alpha})\right)$$

where $\kappa = \lambda^{1-\alpha} 2g(T)\dot{f}(T)$.

[1] Hashorva E.// Elect. Comm. in Probab. —2005. — **10**.

[2] Lifshits M., Shi Z.// Bernouli. — 2002. — **8**, N 6.