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A Blaschke-type bound for a class of unbounded analytic functions

The Blaschke-type condition for zeros z_n of an analytic and bounded in the unit disk function has the form

$$\sum_{|z_n|<1} (1-|z_n|) < \infty.$$

There are many generalizations to unbounded functions (M.M. Djrbashian, F.A. Shamoyan, and many others). Corresponding weights always depend on $(1-|z|)$.

Now let E be a closed subset of the unit circle $\{|\zeta|=1\}$ such that for some real α the integral

$$\int_0 \frac{\mathrm{mes}\{\zeta : \, |\zeta|=1, \, \mathrm{dist}(\zeta, E) < t\}}{t^{\alpha+1}} \, dt$$

converges, and f be an analytic function in the unit disk such that $|f(0)| = 1$ and $|f(z)| \le \exp(D \mathrm{dist}(z, E)^{-q})$. We find the condition for zeros z_n in the form

$$\sum_n (1-|z_n|) \mathrm{dist}(z_n, E)^{(q-\alpha)_+} \le D \, C(q, E).$$

The natural setting is the class of subharmonic functions v and their Riesz measures (generalized Laplacians) $\mu = (1/2\pi)\triangle v$. The results are optimal.

Application

Let U be a unitary operator in the Hilbert space with the spectrum in the proper closed subset E of the unit circle, and T be a contraction linear operator such that $U-T$ belongs to the Schatten — von Neumann class S_q, $q > 0$. Then the sum over all eigenvalues z_n of T in the unit disk

$$\sum_{|z_n|<1} (1-|z_n|) \mathrm{dist}(z_n, E)^{(q-\alpha)_+}$$

is finite.