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A Random Version of Filipov and Bogoliubov Theorems for Hyperbolic Functional Differential Inclusions

Throughout the paper, a, b are two positive real numbers; $Q_{a,b}$ is the rectangle $[0,a]\times[0,b]$ with the Lebesgue σ - algebra.

Denote by (Ω, U, P) a complete probability space, $C^*\left(Q_{a,b}, R^n\right)$ the space of all absolutely continuous (in a Carathéodory sense) maps from $Q_{a,b}$ into R^n, $ACC\left(\Omega\times Q_{a,b}, R^n\right)$ the space of all measurable maps $w:\Omega\to C^*\left(Q_{a,b}, R^n\right)$. Let $\Psi:\Omega\times C^*\left(Q_{a,b}, R^n\right)\to cdL^1\left(Q_{a,b}, R^n\right)$ be a set-value map with non-empty closed and decomposable values in $L^1\left(Q_{a,b}, R^n\right)$.

Theorem. Assume that

1) the multifunction $\omega\to\Psi(\omega, z)$ is $\left(U, B\left[L^1\right]\right)$ - measurable;

2) there exists a measurable map $k:\Omega\to R^+$ such that for every $\omega\in\Omega$, u, $v\in C^*\left(Q_{a,b}, R^n\right)$ and every $(x,\, y)\in Q_{a,b}$

$$h_{L^1\left(Q_{xy}, R^n\right)}\left(\Psi(\omega, u), \Psi(\omega, v)\right)\le\int_0^x\int_0^y k(\omega)\left\|u(s,r)-v(s,r)\right\|dsdr;$$

3) for any measurable $w:\Omega\to C^*\left(Q_{a,b}, R^n\right)$ there exists a measurable map $\rho:\Omega\to L^1\left(Q_{a,b}, R^+\right)$ such that for every $(x,y)\in Q_{a,b}$

$$d_{L^1\left(Q_{xy}, R^n\right)}\left[w_{xy}(\omega), \Psi(\omega, w(\omega))\right]\le\int_0^x\int_0^y\rho(\omega)(s,r)\,dsdr,$$

$$w(\omega)(x,0)=\alpha(\omega)(x),\;\; w(\omega)(0,y)=\beta(\omega)(y),\;\; \alpha(\omega)(0)=\beta(\omega)(0),$$
$$\omega\in\Omega,\;\; \alpha:\Omega\to AC([0,a], R^n),\;\; \beta:\Omega\to AC([0,b], R^n).$$

Then for every $\beta>0$, there exists a random solution $z:\Omega\to C^*\left(Q_{a,b}, R^n\right)$ of the problem

$$z_{xy}(\omega)\in\Psi(\omega, z(\omega)),\; z(\omega)(x,0)=\alpha(\omega)(x),\; z(\omega)(0,y)=\beta(\omega)(y),$$

such that for every $(\omega, x, y)\in\Omega\times Q_{a,b}$

$$\left\|z(\omega)(x,y)-w(\omega)(x,y)\right\|\le\int_0^x\int_0^y e^{k(\omega)(x-r)(y-s)}\rho(\omega)(s,r)\,dsdr+\beta e^{k(\omega)xy}.$$

If in addition there exists

$$\lim_{\substack{M\to\infty\\ N\to\infty}}\frac{1}{MN}\int_0^M\int_0^N\left\{\int_\Omega\Psi(\omega, z)\,dP(\omega)\right\}dxdy=\overline{\Psi}(z),$$

then we can prove a basic Bogoliubov theorem of the method of averaging for inclusion

$$z_{xy}\in\varepsilon^2\Psi(\omega, z),\,(x,y)\in Q_{L\varepsilon^{-1}, L\varepsilon^{-1}}.$$