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Free commutative subsemigroups in topological commutative semigroups

A subset of a topological space is said to be nowhere dense if the interior of its closure is empty. A subset is called meagre if it is a countable union of nowhere dense subsets. The complement of a meagre set is called co-meagre. The well-known Baire Category theorem (see for example [1]) says that a complete metric space is not a meagre set.

Let S be a topological semigroup. We will consider all products of topological semigroups as topological spaces with respect to product (Tykhonov) topology.

The commutative semigroup S is said to be almost commutative free if for each $n \geq 2$ the set

$$\{(s_1, \ldots, s_n) \in S^n | \{s_1, \ldots, s_n\} \text{ freely generates a free commutative subsemigroup of } S\}$$

is not meagre and is co-meagre in S^n.

Theorem 1. *Let S be a complete metrizable topological commutative semigroup. If S contains a dense free commutative subsemigroup then S is almost commutatively free.*

Corollary 1. *The semigroup $(\mathbb{R}, +)$ contains a dense free commutative subsemigroup and hence is almost commutatively free.*

[1] J. L. Kelley. General topology — Birkhauser, 1975, 298p.