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On the weakly nonlinear and symmetric periodic systems at resonance

We investigate in [1] the existence of periodic and symmetric solutions of weakly nonlinear ordinary differential equations at resonance and establish conditions, under which those solutions are either stable or hyperbolic. We present examples to illustrate our theory and also to show advantages of our method to the classical averaging theory [2, 3, 4, 5].

We consider non-autonomous resonance systems of the form

$$\dot{x} = \varepsilon f(x,t), \quad x \in R^n,\, t \in R \tag{1}$$

with a small parameter $\varepsilon \in R$, and a function $f \in C^0(R^n \times R, R^n)$ is symmetric in x and pT-periodic in t, i.e. it holds $Af(x,t) = f(Ax, t+T)$, where $A : R^n \to R^n$ is a linear map such that $A^p = I$ for some $p \in N$, $1 \in \sigma(A)$, where $\sigma(A)$ is the spectrum of A. The case $1 \notin \sigma(A)$ is studied in [6, 7] where unique symmetric and periodic solutions are shown.

The results presented here are generalizations of achievements for anti-periodic problems with $A = -I$ [8, 9], and continuations of [6, 7, 10]. Doubly symmetric solutions of reversible systems are studied in [11]. Symmetric properties of periodic solutions of nonlinear nonautonomous ordinary differential equations are studied also in [12]. Symmetric Hamiltonian systems at resonances are studied in [13].

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