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On convergence of the solutions of a differential equation with two delayed terms

In this contribution we deal with asymptotic behavior of solutions to a linear homogeneous differential equation containing two discrete delays

$$\dot{y}(t) = \beta(t)[y(t-\delta) - y(t-\tau)] \tag{1}$$

as $t \to \infty$. In (1) we assume $\delta, \tau \in R^+ := (0, +\infty)$, $\tau > \delta$, $\beta : I_{-1} \to R^+$ is a continuous function, $I_{-1} := [t_0 - \tau, \infty)$, $t_0 \in R$. Set $I := [t_0, \infty)$. The symbol " $\dot{}$ " denotes the *right-hand* derivative. Similarly, if necessary, the value of a function at a point of I_{-1} is understood as the value of the corresponding limit from the right. The main results concern the asymptotic convergence of all solutions of Eq. (1). We especially deal with the so called critical case (with respect to the function β) which separates the case when all solutions are convergent and the case when there exist divergent solutions. Proofs of the results are based on the comparison of solutions of (1) with solutions of an auxiliary inequality which formally copies Eq. (1).

Theorem *Suppose there exists a function $\varepsilon : I_{-1} \to R^+$, continuous on $I_{-1} \setminus \{t_0\}$ with at most the first order discontinuity at the point $t = t_0$ satisfying $\int^{\infty} \varepsilon(s)\beta(s)ds < \infty$, and the inequality*

$$\varepsilon(t) + \exp\left[-\int_{t-\tau}^{t} \varepsilon(s)\beta(s)\,ds\right] \geq \exp\left[-\int_{t-\delta}^{t} \varepsilon(s)\beta(s)\,ds\right]$$

on I. Then the initial function

$$\varphi(\theta) = \exp\left[\int_{t_0-\tau}^{t_0+\theta} \varepsilon(s)\beta(s)\,ds\right],\ \theta \in [-\tau, 0]$$

defines a strictly increasing and convergent solution $y(t_0, \varphi)(t)$ of (1) on I_{-1} satisfying the inequality

$$y(t) \leq \exp\left[\int_{t_0-\tau}^{t} \varepsilon(s)\beta(s)\,ds\right]$$

on I.

Theorem *The following three statements are equivalent:*
a) *Eq.* (1) *has a strictly monotone and convergent solution on I_{-1}.*
b) *All solutions of Eq.* (1) *defined on I_{-1} are convergent.*
c) *Inequality* (1) *has a strictly monotone and convergent solution on I_{-1}.*

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