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On Representation of Solutions of Linear Differential and Discrete Systems with a Single Delay and its Application to Controllability Problems

For integers $s \leq q$ we set $Z_s^q := \{s, s+1, \dots, q\}$. We derive formulas for solutions of linear discrete systems

$$x(k+1) = Ax(k) + Bx(k-m) + f(k),$$

where $m \geq 1$ is a fixed integer, $k \in Z_0^\infty$ is an independent variable, $x\colon Z_{-m}^\infty \to R^n$ is unknown solution, with constant $n \times n$ matrices A, B, $AB = BA$, $\det A \neq 0$ and with a given vector $f\colon Z_0^\infty \to R^n$.

We also investigate discrete controlled systems

$$\Delta x(k) = Bx(k-m) + bu(k),$$

where $m \geq 1$ is a fixed integer, $k \in Z_0^\infty$ is an independent variable, B is a constant $n \times n$ matrix, $x\colon Z_{-m}^\infty \to R^n$ is unknown solution, $b \in R^n$ is given nonzero vector and $u\colon Z_0^\infty \to R$ is input scalar function.

Finally, we consider the system of delayed linear differential equations of second order

$$y''(t) + \Omega^2 y(t-\tau) = bu(t)$$

where $y\colon R_+ := [0, \infty) \to R^n$ is an unknown vector, Ω is an $n \times n$ constant regular matrix, $b \in R^n$, $b \neq \theta$ is a given vector, θ is a null vector and the control $u\colon R_+ \to R^n$ is a given vector-function, $t \in [-\tau, 0]$, $\tau > 0$, and an initial problem

$$y(t) = \varphi(t), \quad y'(t) = \varphi'(t),$$

and $\varphi\colon [-\tau, 0] \to R^n$ is twice differentiable. Special matrix functions are defined: the delayed matrix sine and the delayed matrix cosine. These matrix functions are applied to obtain explicit formulas for the solution of the initial problem and a controllability criterion.

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