Vasily Chernecky (Department of Higher Mathematics, Odessa State Academy of Refrigeration, Odessa, Ukraine)

Projection Methods for Solution of Fundamental Equation of Risk Theory

Let $F(u)$ be the distribution function of claims $Y_j(=Y) > 0$ with expectation $EY_j = \mu$, $K(u)$ be the distribution of waiting time $T_j(=T) > 0$ with expectation $ET_j = 1/\alpha$, and $c > \alpha\mu$ be the gross premium rate, $j \in \mathbf{N}$. Random variables Y_j and T_j are supposed to be mutually independent. The non-ruin probability of an insurance company, $\varphi(u)$, with initial capital u satisfies the Feller-Lundberg integral equation [1],

$$\varphi(u) - \int_0^\infty dK(v) \int_0^{u+cv} \varphi(u + cv - z)\, dF(z) = 0, \quad u \geq 0, \tag{1}$$

which is the equation of the Wiener-Hopf type. We are interested by the solution $\varphi(u)$ which is a monotone nondecreasing function of u, satisfying the condition

$$\varphi(u) \nearrow 1 \quad \text{when} \quad u \to +\infty. \tag{2}$$

Exact integration of the problem (1)-(2) presents difficulties and in the majority of cases may be done only by numerical methods. Using the results of the works [2]-[4], the applicability of the projection methods to the solution of the problem (1)-(2) is justified. Number of illustrative examples are given.

[1] Grandell J., Aspects of Risk Theory, Springer-Verlag, 1991.

[2] Gohberg I.C., Levchenko V.I., On the convergence of the projection method for solving the degenerate discrete Wiener-Hopf equation, Mat. Issled., VI, no. 4, 20-36(1971) (Russian).

[3] Gohberg I.C., Levchenko V.I., On the projection method for the degenerate discrete Wiener-Hopf equation, Mat. Issled., VII, no. 3, 238-253(1971) (Russian).

[4] Pomp A., Über die Konvergenz des Galerrkischen Verfahrens für Wiener-Hopfsche Integralgleichungen in den Räumen L^p, Math. Nachr. 87, 71-92(1979).